Point, Line, Plane:

Basic Elements of Formal Composition in Bauhaus and Shape Computation Theories

By

Sotirios D. Kotsopoulos

Master of Architecture
Southern California Institute of Architecture, 1995

SUBMITTED TO THE DEPARTMENT OF ARCHITECTURE IN PARTIAL FULFILLMENT OF THE REQUIRMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN ARCHITECTURE STUDIES
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
JUNE 2000

Signature of Author:
_____
Department of Architecture
May 18, 2000

Certified by:
_____
Terry Knight
Associate Professor of Design and Computation
Thesis Supervisor

Accepted by:
_____
Roy Strickland
Principal Research Scientist in Architecture
Chairman, Committee for Graduate Students

Readers:

George Stiny
Professor of Design and Computation

William J. Mitchell
Professor of Architecture and Media Arts and Sciences
Dean, School of Architecture and Planning

Point, Line, Plane:

Basic Elements of Formal Composition in Bauhaus and Shape Computation Theories

By

Sotirios D. Kotsopoulos

Submitted to the Department of Architecture on May 18, 2000 in Partial Fulfillment of the Requirements of the Degree of Master of Science in Architecture Studies

ABSTRACT

Architecture is not representational. It does not stand for something else. However, the process of its formation is inclusively dependent upon a series of dynamic graphic calculations that result into a series of spatial descriptions. This process can be equated to a non-linear sequence of computations with points, lines, planes, and solids, on the plane and in physical space. This study examines the functional and perceptual properties of points, lines and planes. How do basic elements behave in formal composition, and how do computations of form affect basic elements? The context of the study is composite. Shape computation theory that involves algebras of basic elements and shape rules provides a flexible and expressive computational apparatus, while the systematic approach of the Bauhaus on nonrepresentational composition, and the theories of P. Klee and W. Kandinsky in particular, provide artistic insight at a perceptual and interpretational level.

Thesis Supervisor: Terry Knight
Title: Associate Professor of Design and Computation

to Maria

ACKNOWLEDGEMENTS

CONTENTS

INTRODUCTION

III.   PLANE
       A.   Mathematical concept
       B.   Graphical concept
       C.   Dictionary
              1.   Planar forms
              1.1   Interaction of planes
              1.2   Actual and perspective planes
              1.3   Activation of planes
              2.   Graphic context
       D.   Basic elements on the graphic context
              1.   Line on plane - Relation to the boundaries
   2.   Plane on plane

CONCLUSIONS

REFERENCES

INTRODUCTION

Architecture is not representational. It does not stand instead of something else. However, the process of its formation is inclusively depended upon a series of dynamic graphic calculations that result to a series of spatial descriptions. This process can be equated to a non-linear sequence of computations with points, lines, planes, and solids, on the plane, and the physical space. Basic elements such as points, lines, and planes, arranged on the plane, compose two-dimensional drawings, while planes and solids arranged on the physical space, compose three-dimensional models. Basic elements and their combinations have a profound value. They are abstract and inclusive. Without intrinsic structure, they allow us to attribute structure and meaning, as we want. In design, dynamic figuration proceeds from the general and abstract to the particular and concrete. This study examines the functional and perceptual properties of points, lines and planes. How do basic elements behave in formal composition, and how do computations of form affect basic elements? The context of the study is composite. Shape computation theory that involves algebras of basic elements and shape rules provides a flexible and expressive computational apparatus, while the systematic approach of the Bauhaus on non representational composition, and the theories of P. Klee and W. Kandinsky in particular, pro-vide artistic insight at a perceptual and interpretational level. The reason for attempting this linking is threefold. First, because all three approaches are systematic and focus on non-representational composition. Second, because they are not concerned with the artefact as a material object, but with the manner it is produced, not with form as an immutable value, but with formation as a process. Third, because the examination of the properties of basic elements is a step towards the essential, towards the functional as opposed to the impressional. We learn to look down on formalism, and to avoid taking over finished products. This should prevent us from regarding artefacts or designs as something rigid, fixed, or unchanging. After all, the world in its present configuration is not the only world possible!

My interest in basic elements begins from the observation that verbal description is not enough to explain architectural form and the process of formation. Objects demand visual description. However, visual description as fixed representation does not say enough either. If we are interested in form we need to approach it dynamically, as a process. Architecture does not build images, and moreover it does not need to produce structures in agreement to a predefined mode of expression. It introduces new instances into an already existent language, the architec-tural language. This becomes evident in the experience of a significant work, which usually resembles a discovery, it is sudden and unforeseen, and it is not immediately available for description. In both architectural and computational terms it is not interesting to have to deal with fixed descriptions. Therefore we need to develop a flexible and dynamic computational

process. The process is computationally ambiguous since there is no golden rule to predefine what is better, but only the empirical flux of things. Basic graphic elements in their abstraction constitute the elementary visual qualities that we use in this abstract process. Points, lines, and planes manipulated on the plane become immaterial expressions of our spatial ideas. All basic elements and their arrangements may be approached in unanticipated ways. What becomes essential is that graphic calculations follow our perception, independently from what this may cause to description.
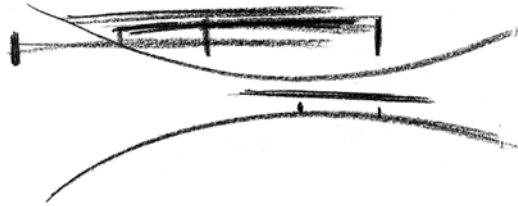

T. Ando

The idea of an abstract process of composition in the arts is old. Aristotle (Poetic 1459 A 19) referring to the process of synthesis, notes that "the composition does not have to be similar with something existing in the real world, but it should be approached as an individual singular action that completes itself". In the first half of 15th century Alberti introduced a fundamental, as well as useful, abstraction. He distinguished the conceptual process of design of an artefact from its material manifestation. Alberti warns that the architect must know exactly what to do before construction can begin. The building must already be fully complete in his mind. Accordingly he defines design as "a firm and graceful preordering of the *lines* and *angles*, conceived in the *mind*, and contrived by an ingenious *artist*" (1955, p. 1-2).

Paul Valery, (1924, p. 58) observed a similarity between architecture and music: both are abstract and non-representational, because they are making us see things other than themselves. They do not imitate but they force matter to take conceptual forms. They give form to rules and they take form by rules. The explanation to this mystery can be found in their affinity to those half –abstract, half-concrete beings we call shapes. According to Valery "shapes have impact in vision, in touch, in hearing but also in thought, and in number". He defines shapes as combinations of lines, where lines are the image of an "unreasonable oddity" without beginning, without end, and without any other meaning except that of freedom. The architect organizes lines in shapes geometrical or not, by using rules. Finally the rules are revealed to the viewer through form. For the sense of harmony that architecture causes to the viewer, the viewer responds actively with an infinite perceptual multiplicity that is constructed by the eye, without the slightest effort. This supreme delight is forced by the awareness of the calculated shapes, and the organized dimensions that force perception to build infinite imaginary relations.

Klee (1956, p. 24) declared that "art does not reproduce the visible but makes visible". For Klee "the very nature of graphic art lures us to abstraction". Combinations of graphic elements produce forms that express the schematic quality of the imaginary with great precision. The more emphasis the graphic work puts on the basic formal elements, the less well suited is to representation. He sets a distinction between basic visual elements, without which a work in any art cannot come into existence, and secondary elements. Formal elements of graphic art are "points, and linear, planar, and spatial energies".
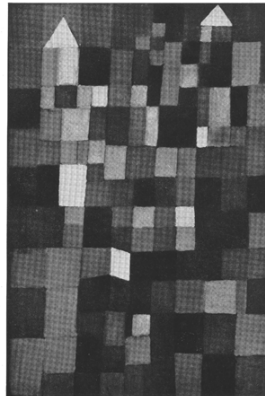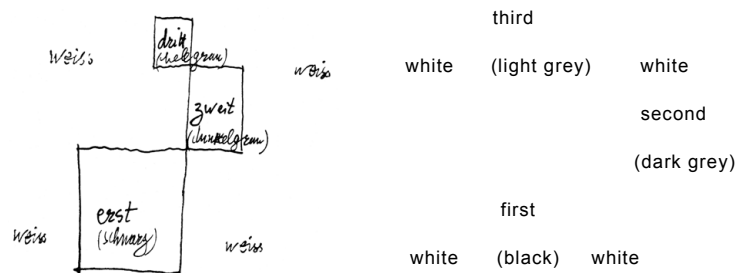
Klee "Architecture"   Compositional rule for the contrast-relation (Klee)

The composition of elements proceeds according to rules. Klee rejects "prefiguration" and sees the process of *formation* (*Gestalltung*) dynamically. *Formation* rather than the *form*, becomes the primary aim of art. *Formation* is based on a world of diversity where elements are moving self evidently, in a state of primal motion and rule application causes momentarily development and fixation. For Klee "to be abstract means to distil pure formal relations". The creative powers, which are named "perception" and "formation", can also be expressed as "perception rules" and "formation rules". Accordingly, the "Theory of Formation" proceeds to the study of perception and formation rules and the study of basic elements. Klee's approach to graphic art struggles for systematic precision, but there is an equal effort to avoid the limitations of uniformity. Logic and calculation, which determine the mechanism of generative and productive techniques, must be developed into more subtle and penetrating procedures harnessing action and knowledge, manual and mental activity. The formalism that Klee proposes is based on rationality without numerical formulae, but rooted in experience.

Kandinsky (1926, p. 20) suggested that his primary concern is "the role of the artist in conscious creation". The artist should not wish to initiate "appearances". Abstraction is opposed to empathy and scientific approach is proposed as a method of artistic inquiry. The underlying theme is the discussion on basic elements and the need for a theory for creation and interpretation of art. "The progress achieved by systematic research will give birth to a dictionary of

elements that, developed further, will lead to a "grammar" and finally to a theory of composition. The investigation proceeds to the examination of points, lines and planes, while emphasis is given on methods of progressing from analysis to synthesis. Kandinsky employs the parallel concept of "language", the "graphic language" has its own evolving laws and grammar. There is also a profound interest in mathematical expression, which aims to establish an analytical basis for art theory. The interest in mathematical expression tends in theoretical and practical directions. In the theoretical approach *logic* plays the most important role, while practice is purely purposive. Here, *logic* is subordinated to purpose, so that the work attains the highest quality.
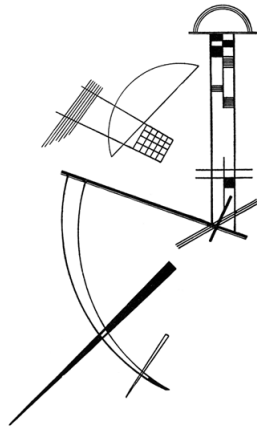
Kandinsky: Geometric composition

Kandinsky worked almost exclusively with designers and architects, attempting to construct and justify methods of teaching that would relate the 'immaterial' and the 'material' in the creation of a new artistic realm. The study of form includes study of basic elements and study of rules that Kandinsky names "developing attitudes". The notion of "basic design" and "basic course" (*Formlehre*) are explicitly introduced as educational methods for designers. Kandinsky states that "the form of the work and its elements are the material for analysis, and not the psychology of the creation, nor the psychology of aesthetic perception, nor the historical, cultural, sociological or other problems of art".

When Paul Valery said: 'I did not want to talk, but to "construct"', he was expressing the preference of a modern poet. Before him, poets like Malarme saw the existence of a constructive rule as precedent to the poetic sentence. Cubists, poets, painters, and artists confronted this dilemma: how to ordain the content to the provisos of a "tectonic" composition of form and color. The concept that makes these ideas similar with the systematic teaching of Klee and Kandinsky is *construction* according to rules. *Construction*, in its architectural sense, underlies the approach of the sculptural and pictorial composition at the Bauhaus. Klee and Kandinsky isolate basic formal elements that underlie all visual expression. Their theoretical writings are concerned with the rules that govern the distribution and interaction of those elements.

Kandinsky

Klee
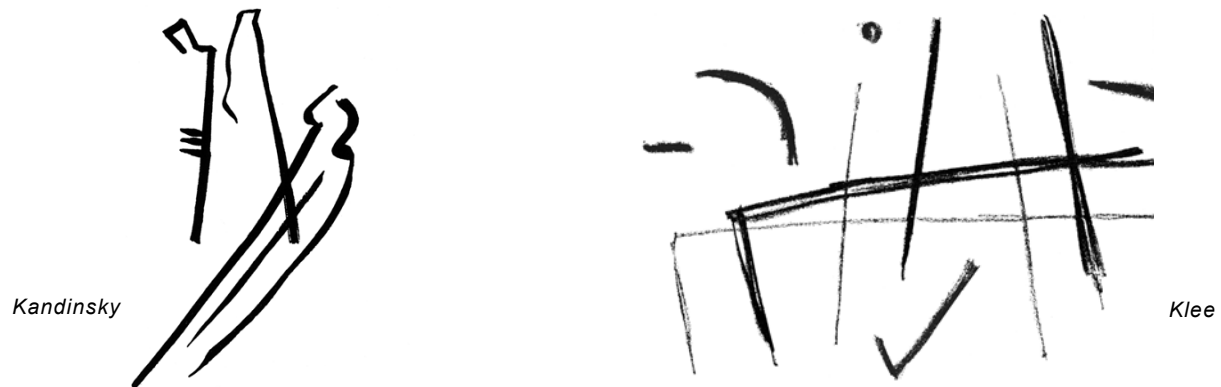
It is not an oxymoron that as symbolic analytic processes support a growing fraction of human activities, the creative and visual ones look more and more obscure and incomprehensible. In the past decade new powerful descriptive and generative approaches in design have been developed in parallel with the introduction of computer aids in the design process. Today as the computer influences deeply the way we think and practice architecture we need to address the questions of creative processes in a new way. An additional reason is that a rigorous computational approach may open new directions to design. But neither the perspective of radical transformation of architectural practice nor the change of architectural education is a reason to disconnect us from the original ideas that the knowledge and experience of the discipline of architecture offers.

Shape grammar theory aims, amongst others, to preserve the autonomy, the ambiguity and the creative potentiality of the design process, and to incorporate the qualitative character of formal composition in computational processes. A parallel goal of the grammatical theory is the rigorous analytic and synthetic approach of "style" through the development of flexible formalistic apparatuses such as that of "grammars" and "design languages". In analysis "grammars" investigate the compositional methods of an existent style. The analytic approach does not examine the work as something fixed. Exercises of this type have as main goal to introduce sets of rules that can describe expressively a specific style. In synthesis, the artist just like the scientist sets out to find a solution to a spatial problem. These innovations can be applied again towards the solution of other spatial problems, much like variations to a theme. In these variations the artist recognizes a personal style. Similarity does not mean superficial copying but only preference to a set of rules.

In shape grammars basic elements such as points, lines, planes, and solids and combinations of those are organized according to their dimension in algebras and are manipulated according to rules. As has been defined in Stiny (1991), an algebra U_{ij}, contains shapes. Each shape is a

finite or possibly empty set of *basic elements* that are *maximal* with respect to one another. *Basic elements* are defined in dimension $i=0,1,2$ or 3 and manipulated in dimension $j > i$. In these algebras shapes can interact in computations that may occur in any dimension. The shape algebras provide the mathematical machinery for shape computations. Shapes can be compared, manipulated, and combined in all possible ways. Operations such as sum, difference, and similarity transformations, and relations such as the *part of*, which allows on shapes parts to be embedded in one another, facilitate calculations with shapes in all dimensions. A shape rule A→B applies on a shape C in two steps. In the first step the rule "recognizes" any part of C that much to any transformation of A. In the second step subtracts this part from C and adds the same transformation of B in its place.

$$U_{00} \quad U_{01} \quad \mathbf{U_{02}} \quad U_{03}$$

$$U_{11} \quad \mathbf{U_{12}} \quad U_{13}$$

$$\mathbf{U_{22}} \quad U_{23}$$

$$U_{33}$$

C

$t(A) \leq C$
$(C - t(A)) + t(B)$
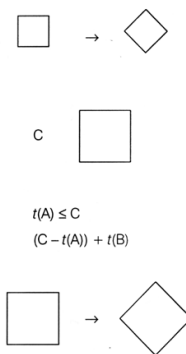
The U_{02}, U_{12}, U_{22} algebras combined with symbols, that we call labels, and attributes of texture and color, that we call weights, can express computationally all the activity that takes place on the drafting table. The elements here are points, lines and planes that are manipulated on the plane. The operations give us the computational tools to erase a shape, or a line, to add one shape to another but also to implement computationally notions such as rhythm, repetition, etc. The transformations extend our perception on the spatial properties of shapes and the different ways that they can be arranged on the plane. Rules provide the expressive tool to relate spatially one shape with the other with the immediacy that we think, from the very early stage when the first lines are executed to the very end of the composition.

This study has three goals. First, to look at a historic precedent computational design theory and expose the similarities and the differences with shape grammars, from technical and perceptual points of view. Second, to show that our design intuitions are inhabited to a large degree by the ambiguous visual properties of basic elements, and to exhibit how these can be effectively handled computationally by the algebras of design. Third to look with analytic interest, and from an educational viewpoint at the specific expressive basic visual vocabulary of the Bauhaus and its corresponding design language as an attempt to deal with design, and design education, computationally.

I.   POINT

A.   Mathematical concept

The first known definition of the point is that given by the Pythagoreans. Proclus (Proclus, p. 95, 21) defines the point as a "monad having position", the only indivisible element *in the subject matter of geometry*. Aristotle (*Metaphysics* 1016 b 24) uses an equivalent definition: something, which is indivisible in respect of magnitude but has no position, is a *monad*, while that which is similarly indivisible and has position is a *point*. For Plato the existence of points as a discrete *genus*, is a geometrical fiction. He defines the point as the beginning or end of a line, and he frequently refers to 'indivisible lines'. Euclid (Book I. 1.) defines the point as that which is indivisible into parts, omitting the necessity that the point must have a position. Aristotle illustrates further his conception: a point is not a *body*, and has no *weight*. He also attempts to explain the transition from the indivisible, or infinitely small, to the finite or divisible magnitude. It is clear that no accumulation of points however far it may be carried can produce anything divisible, whereas a line is a divisible magnitude. Then, it holds that points cannot make up anything continuous like a line, a point cannot be continuous with another point, and a line is not made up of points. A point, Aristotle explains, is like *now* in time. *Now* is indivisible and is not a part of time, it is only the beginning or end, or a division, of time. Similarly, a point may be an extremity, beginning or division of line but is not part of it or of magnitude. It is only by *motion* that a point can generate a line (De anima I. 4, 409 a 4) and thus be the origin of magnitude. In modern geometry, the effort to define the point *a priori* is not made. The mathematician Max Simon observes (*Euclid*, p. 25) that "the notion 'point' belongs to the limit notions, the necessary conclusions of continued, and in themselves unlimited, series of presentations. The point is the limit of localization. If this is more and more energetically continued, it leads to the limit-notion 'point', or better 'position'. Content of *space* vanishes, relative *position* remains". Then, 'point' is the extremest limit of that which we can still think of, but not observe as a *spatial* presentation, where not only does extension cease but even relative place, and in the Euclidean sense any 'part-notion' becomes nothing.

B.   Graphical concept

In their approach to basic elements, Kandinsky and Klee adopt many of the Pythagorean conceptions, but they broaden mathematical formalization to treat points, lines, and planes, as needed in their graphic computations. The point preserves its fundamental geometrical properties of indivisibility and position, but its incorporeal conception is opposite to the essentiality of pictorial expression. For a painter like Kandinsky (1924, p. 25) the geometric point considered in terms of substance equals zero. However, the point in the graphic arts takes material form and reason of existence: it is the result of the initial collision of the tool with the material plane.

Kandinsky departs from the geometric conception of the bodiless and weightless point, to define the corporeal "graphic point". He identifies it as the "briefest" or "smallest" form, the more sharply defined unit of pictorial expression, or the proto-element of painting. The invisible and indivisible geometric point still mirrors its symbolic significance to the graphic point but in its actual graphic use the point becomes a sharply defined, indivisible shape. Its boundaries enclose the innermost concise form of graphic expression.

The beacon guiding Klee through his adventures in seeing, is the line. He adopts the exegesis of Proclus that the point in itself is "negative", or "dead", because it denies all dimensions, and he makes use of the Aristotelian example to associate the "concept-point" with that of an instant (1956, p. 494). However, always authentic to his dualistic convictions, Klee never accepts a concept in itself: a concept is not thinkable without its opposite. Concepts appear in pairs. Then, when he determines the alternate forces of the design procedure as "rest and unrest", the point plays the role of an agent at rest, while mobility is the condition of change (1956, p.19). The weighted point is not invisible, but an infinitely small planar element. The self-moving point introduces the first dimension by generating a line, just as a line comes into being after the application of a pencil on the graphic surface.The abstract geometrical conception of the point demands that is a 0-dimensional element, a definite indication of position. The graphic point is a visible but non-divisible small shape. The point is conceptually predefined as a symbol although graphically acquires form, texture, and size. This follows the necessity that each point corresponds to exactly one position. As a graphic form the point appears equally disjoined from all sides, its fusion with the surrounding forms is minimal, and seems non-existent in the case of perfected roundness.

The expressive minimalism of the point and its strong concentric qualities can still find use in the arts and in architecture. Shape grammars accommodate all the different abstract properties and possible practical utilities of points. A shape algebra U_0 contains 0-dimensional geometric points and shapes that are arrangements of points. We can manipulate these points in 0,1,2 and 3 dimensions. In U_{00} a single point is the only shape of the algebra, while in U_{01}, U_{02}, and U_{03} shapes are finite arrangements of points manipulated in the line, the plane, or in three-dimensional space. Correspondingly, an algebra V_0 (V_{00}, V_{01}, V_{02}, V_{03}) contains 0-dimensional labeled points that can take any form and responds to the graphic demand for symbols of various forms, while a W_0 (W_{00}, W_{01}, W_{02}, W_{03}) algebra supports all the above additionally with weights, colors, textures and other attributes. Moreover, the algebras can always be combined with others to accommodate more complex graphic demands. But how do points behave?

In the general case shapes are finite arrangements of maximal elements and can be identified uniquely by sets of basic elements without definite parts. In shape computations, the parts of a shape and the possibilities for rearrangement are indefinite, depending on rules and transformations. The parts of any shape made out of lines, or planes may be approached in infinite many ways each time a rule is applied. In the case of indivisible points (U_0) the part relation that allows lines and planes to be embedded in others, to fuse and to separate naturally, holds only as identity. Thus, the finite number of the power set of the subsets of any set of points exhausts all the parts of the shape that is made out of points, and limits the possibilities in finite number of combinations. This idea is exhibited in Stiny 1991. The following example with three shapes: the first contains 7 points (U_{02}), the second 8 maximal lines (U_{12}), and the third three planar shapes a circle, a triangle, and a square. The shape in U_{02} can produce a finite number of combinations between points, that may be seen as arrangements, while the shape in U_{12} and U_{22} contains infinitly many parts, segments of lines or planes. The visual power of the example depends also on the number of points included in the initial shape in U_{02} .
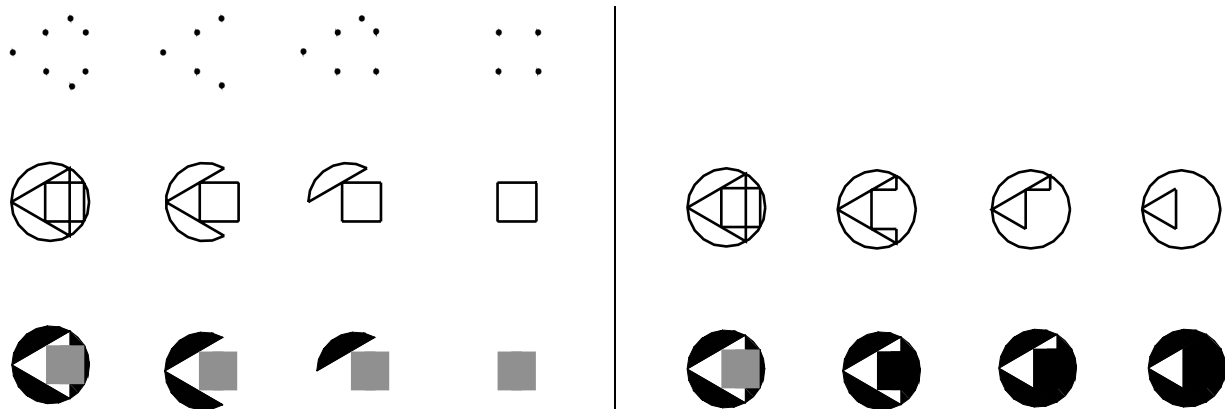
C.   Dictionary

Kandinsky (1924, p. 83) and Klee (1924, p.21) organized an elementary visual dictionary. Their ultimate goal was the creation of a basic "grammar" that would lead to a theory of composition. In this effort they established classes of abstract elements that constitute important formal factors. This basic dictionary extends to points, lines and some of their elementary combinations on the plane, and develops the basis of a "visual language", which cannot be attained with words. Kandinsky sees this basic dictionary as one of a living language, being immutable as it undergoes perpetual changes: " Words become submerged, or die. Words are created, come new into the world; foreign words are brought home from across the borders". Kandinsky and Klee give two kinds of definitions of basic elements and their relations: relative or absolute.

In the first case definitions are relative if they refer to a specific graphic context or to a coordinate system, while in the second they are independent of context. In shape grammar theory, elements are defined only relatively, in relation to a coordinate system. However, both Bauhaus artists suggest rules that do not restrict visual ambiguity, but incorporate perceptual transitions in computations. This can be expressed by transfers of graphic elements from one algebra to another i.e., points, or lines can be seen and manipulated as planes, etc.

1.   Appearance

An essential characteristic of form creation is fusion and mutation of parts that come from diverse elements. The point cannot participate dynamically in computations by changing parts, but only through sum, however, its appearance still can acquire an unlimited variety of forms. For Kandinsky (1924, pp. 30-50) the graphic point depends on:

1.   form

2.   texture

3.   size, and relation to the other forms of the plane

1.1   Form

All graphic elements have some visible form. The abstract point is ideally small and round. But even the minimal point can get other forms, depending on its graphic use. The need for points with various forms comes partialy from its symbolic nature, and has no further implications in the way the point behaves in computations. The point can assume an unlimited number of forms: jagged, geometrical, or develop into entirely free shapes. Different tools in their contact with the graphic plane can produce an unlimited diversity of points. No boundaries can be fixed and the realm of points is unlimited.

Kandinsky

The above concept can be supported computationally in a shape algebra W_0 that contains weighted points. Points of various forms that retain the properties of indivisibility and definite position, and can also be manipulated in all dimensions as symbols in V_0 algebra. The change from a typical round point to one with different form could be expressed in the following rule.

U_{02} W_{02}

1.2   Texture

The texture of an element affects the manner it is visually combined with other elements and with its graphic context. It can be expressed by a variety of different weights, and tones. Weights are used extensively in architecture, and the visual arts, where different thicknesses of graphic elements become a fine medium of expression. Kandinsky (1924, p. 50) observes that texture possibilities should be given even in the limited field of the point. Weights and textures can be handled computationally in a W algebra that can be extended to points lines and planes. In the case of points a W_0 algebra contains weighted points that are manipulated in a line, a plane or in 3-dimansional space.

1.3   Size and relation to other forms

The point grows in size and occupies an area of the plane. The crossing of a size-boundary can make the point appear as a plane. This perceptual transition is emphasized by Kandinsky as an important feature of abstract basic elements. In the case of the point it "reveals the expressive multiplicity of the smallest form attained by slight changes in its size". Moreover, it is unlikely that any numerical expression could possibly inhibit our sensory perception and determine this size-boundary. Kandinsky and Klee steadily avoid the constraints of computational uniformity. A form or element can change dimension at any stage of the synthetic process. The relation to the other forms seems here the only parameter that we can refer. As new elements inform the process it can become necessary that a point should be treated as plane or as linear shape. This remains entirely a matter of personal decision, since no fixed boundary between points and other elements can be found. A weighted dot that has been placed on the empty plane as a point,
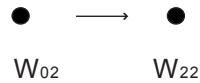
can be seen as a plane, when a very thin line appears next to it.

Kandinsky

Kandinsky (1924, p. 29) introduces the notion of a rule that doesn't seem constructive. In fact it changes the field: the point becomes a planar shape and can be manipulated like any planar element. This visual-computational transition can be expressed in the following shape rule.

$$\bullet \longrightarrow \bullet$$

$$W_{02} \qquad W_{22}$$

The rule substitutes the weighted dot (W_{02}) on the left side, with a weighted plane (W_{22}) on the right side, depending on how we choose to handle the dot. A perceptual shift can be expressed computationally as a transfer from a particular algebra to another. The combination of all three parameters of form, texture, and size, and the perceptual transition of the point to a small plane can be expressed in a W_{22} algebra .

C.   Composition

The examination of elementary arrangements of basic elements with the purpose to reveal their major expressive attributes in relation to some basic context, becomes a standard experimental procedure for Kandinsky. In the case of the point, position and quantity are explored in the predefined context of a square plane.

1.   Centric structure

The positioning of a point at the center of a shape illustrates one of the most common uses of this basic element in all different areas of design. The point acquires the character of a center, and also, reveals the nature of the center itself. In architectural design the manipulation of the center of any given context or spatial arrangement plays a consequential role in the process of composition.
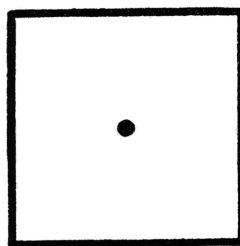

Kandinsky

The concept of the center can be approached more constructively, to produce composite forms. Kandinsky and Klee introduce examples in which the point evolves in concentric, radial, or spiral structures. In the simplest case, computational rules that arrange shapes or planes in relation to an initial center, at a distance *d* or a radius *r*, can be used to generate designs in a

U_{12} or U_{22} algebra. The same concept can produce concentric structures with colors or weights in a W_{22} algebra. The productive growth of the point in concentric waves of geometrical or free forms generates a two-dimensional structure emanating from nuclear strata.

radius r :

distance d :

Klee
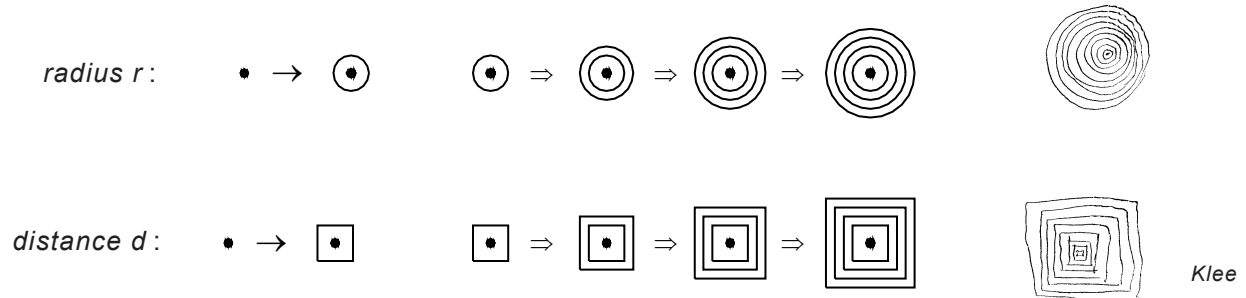
The more characteristic centric structure is the circle. The spiral progress of aline from a center point produces spiral structures which hold different properties from the circle. Rhythmic repetition of any type of line, about a common center creates the radial structure. Any combination of the above centric structures is possible by superimposition of their centers.

Kandinsky
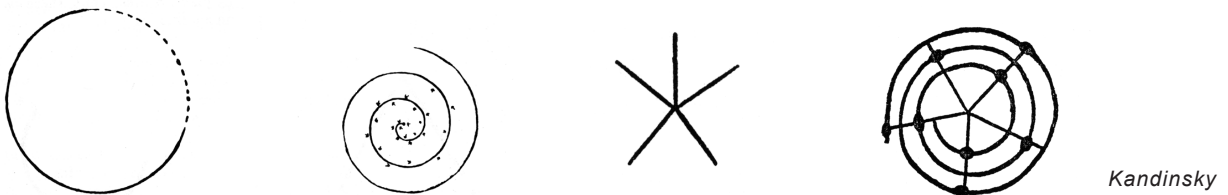
As an extension of the axial concept we can generate through rotation of any linear shape or planar element, about a common center point, the two point groups of symmetry. Rotations about a single point through $2p/n$ generates the cyclic group C_n while, an additional reflection through an axis that passes from the given center generates the dihedral group D_n. Arrangements of this type are widely used in the arrangement of architectural plans (example, F. L. Wright Suntop Homes).

C_4
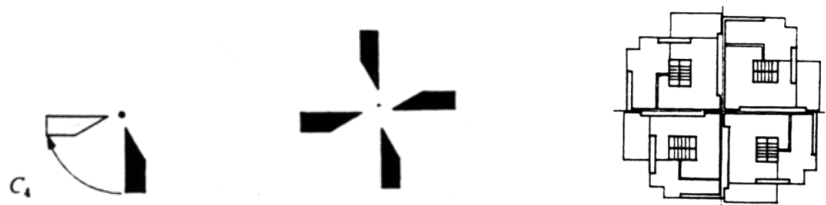
March-Steadman

The above arrangements evolve though computations that are based on the concept of the centric structure. That is, form-production about a center, where the concentric qualities of the point are accentuated and projected in two, and three dimensions. The rules apply in reference to an initial point to generate forms that mirror its abstract concentric qualities. Apparently, each of the centric structures (concentric, radial, spiral) that is generated from an initial center-point, and a distinct set of rules, possess distinct properties. According to the functionalistic artistic approach of Kandinsky and Klee this is not a thing that should be overlooked.

2.   Eccentric Structure

An eccentric structure consists of a point moved from the center of the plane. The arrangement can be seen as the counter concept of the centric arrangement. It transforms the visual impact of the graphic point from the absolute position of the center to a relative position that depends on the relationship with the boundary. More generally, the eccentric point starts to interact with its context. Klee proposes a constructive use of the eccentric point. The displacement of the point can be used in the construction of pictorial depth.The eccentric position of the point effects the nuclear strata of a centric structure, and the density of the boundaries of the cocentric linear or planar shapes creates the illusionistic three-dimensional space. This computation can be executed with lines on the plane in U_{12} algebra, with planes on the plane in a U_{22} algebra, or in a compound U_{12}, U_{22} algebra that includes planes and their linear boundaries. Klee uses the computation often in his paintings by superimposing several eccentric structures that create the visual effect of multiple perspective views on the same canvas.
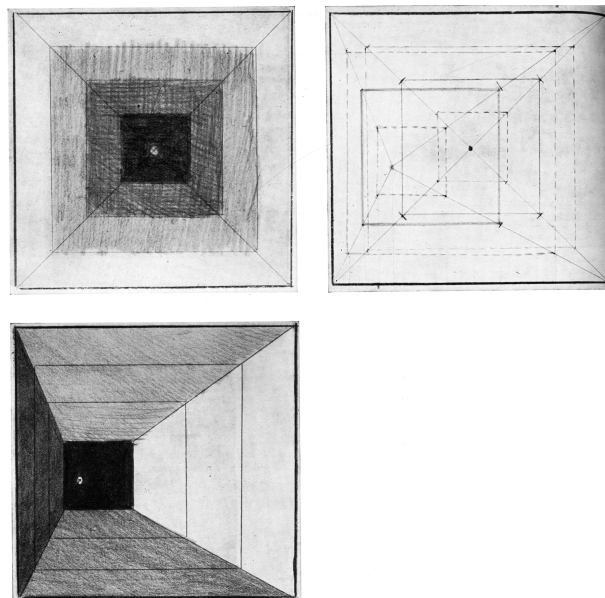
Klee

3.   Quantitative Increase

The repetition of an element produces more complex visual results. Accumulation of points especially in the case that they are not identical can develop a multifaceted graphic effect. Multiplicity and diversity of form, texture, and size, which establishes the graphic point in countless entities with distinct values, is the fundamental expressive medium of the typical graphic techniques like etching, woodcut, and lithography. Properties of this type could be accommodated computationally in a W_{22} algebra, where points can be treated as elementary planes diverse in shape and tone.
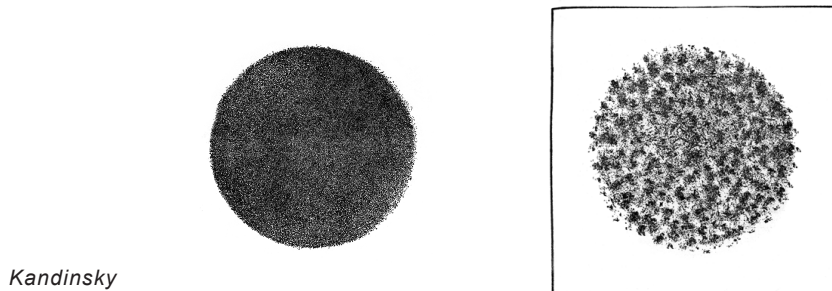
Kandinsky

However, the use of sequence of points with the intention to construct a line is not justifiable for Kandisnky (1924, p. 53) : "This line is condemned to a poverty-stricken half-life". True to his functionalistic approach to basic elements he emphasizes the value of differences as originally purposeful and well-founded. Elements in computations should be treated for what they are.

E.   Points in architecture

Points can also be the medium of subordination since repetition in an ordered fashion is a source of rhythm. In architecture, the organization and interaction of multiple overlapping networks of elements and activities makes the use of overlapping grids a common medium of subordination. Here the point signifies a noteworthy position of an "event" that takes place in the abstraction of the plane or the three-dimensional space. Changes in the graphic size, the form, or the weight of the points may signify corresponding changes in the interpretation. However, the concentric qualities of points remain unaltered since graphic points grow out of their centers.
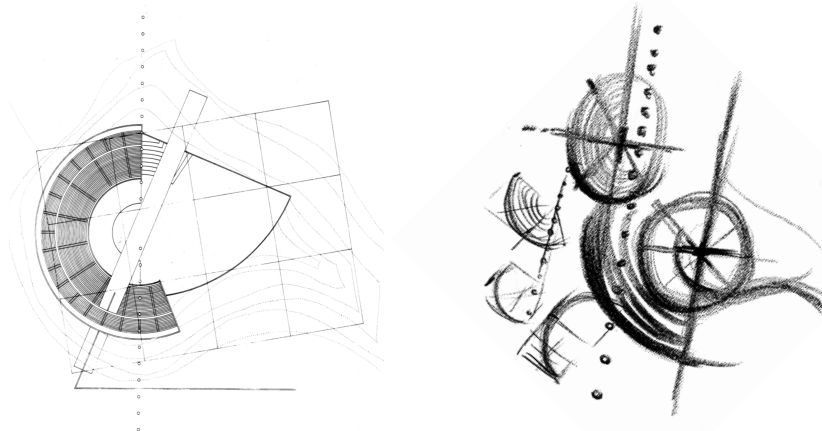

T. Ando

The implications of repetition depend upon the type of points, and the canonic or free mode of repetition. A repetitive series of points of the same weight, arranged linearly, in an architectural ground plan can be seen as an indication of rhythm of form, structural order, and direction of movement. In space the point is the result of cross-section of several planes, or the termination of an angle. The peak of any architectural form can be seen as a point in an elevation.
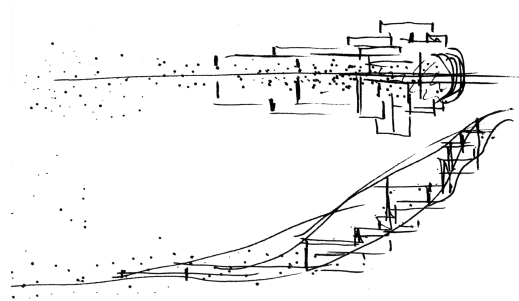

T. Ando

Architectural design occurs in algebras that contain points, lines, planes, and solids. Combinations of algebras like U_{12}, V_{02}, W_{12} and W_{22}, which manipulate lines, labeled points, weighted lines, and weighted planes, are employed to support computations that occur on the drafting table. The action of points can be restrictive or creative. The restrictive action of points in calculations with lines and planes is twofold. First, labeled points (V_{02}) are used as symbols to to assign graphic and symbolic attributes like weights and colors, functions, materials, etc. to lines and planes. Second, weighted round points (W_{02}), or points of various forms, arranged on the plane and manipulated as weighted elementary planes (W_{22}) are used to visualize boundaries of elements, or grids. In this way, points support the articulation of "meaning" or "rhythm", by restraining the free interaction of lines, and shapes thus establishing an underlying structure.
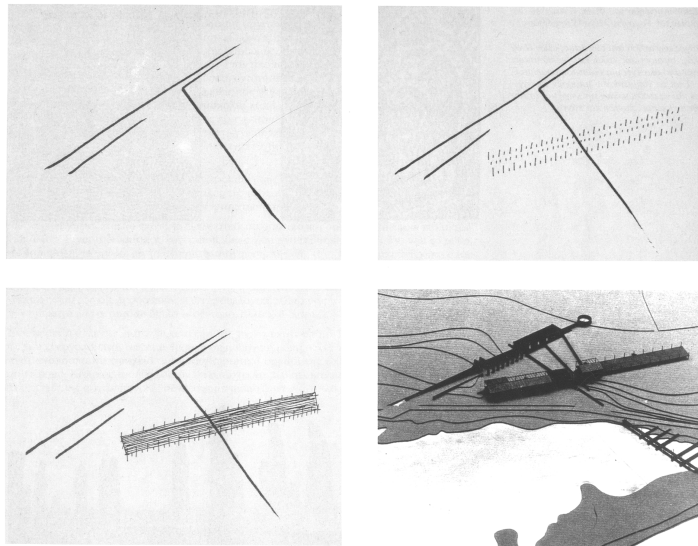
However, we should not disambiguate the action of points, which also has deep creative impli-
cations. The accumulation of points of diverse form that can be suddenly seen and manipulated
as planes, the slight convergence or divergence of ambiguous stains, spots, and dots, their
density or looseness, their weights, the continuous or discontinuous character of their rhythm
etc. Analogous repetitive marks in drawings and models should not be overlooked as symbolic
indications, or side effects of a capricious technique since often they can be traces of the
deepest intentions for the composition.
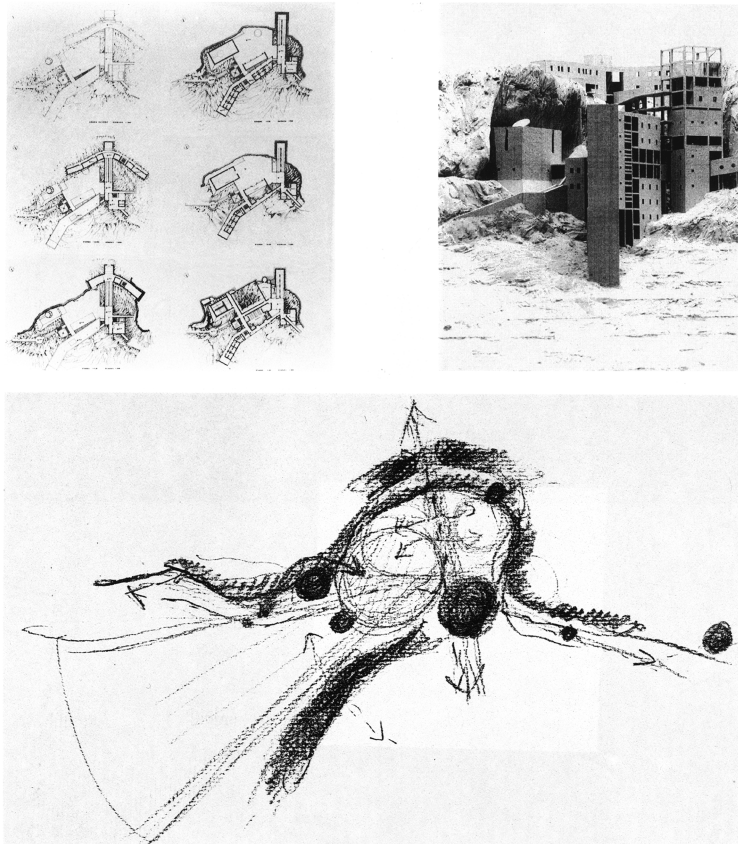
The geometric constraints for points in this case should be overpassed. The graphical character
of the point allows all kinds of points to occur naturally. Points that begin as simple indications
of position demand to express form. They become planes or linear shapes that are manipulated
dynamically and change size, texture, and dimension in ananticipated ways. This ambiguous
interaction of scale, dimension, and parts, between graphic elements becomes a usefull and
expressive tool in composition where the solutions of form emerge graphically and the dimen-
sion of graphic elements cannot be fixed.
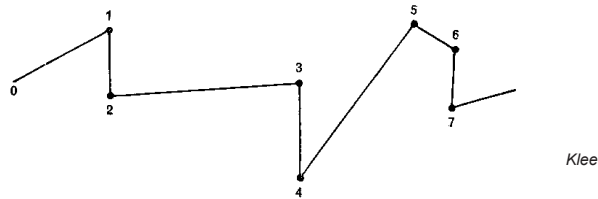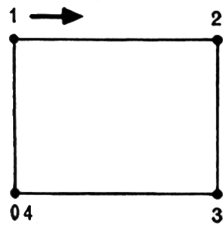
Klee

The restrictive and creative action of points has its foundations in graphic representation. Klee notices that points restrict lines by determining, and visualizing their boundaries. In another illustration, the point discharges its tension towards another point, to generate a line. Points of equal or different intensity that lie on the plane, or others that belong to restricted lines visualize free or restricted rhythms.
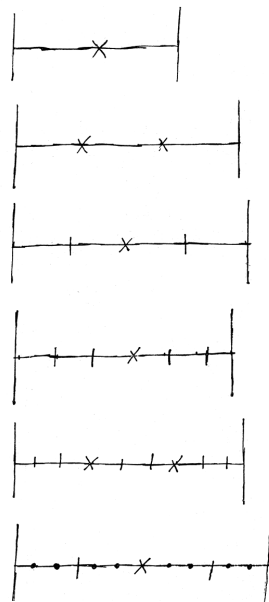
Linear points

Planar points
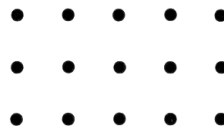
Klee

Klee (1956, p. 152) rejects any notion of predetermined "structure" or "prefiguration" and sees the process of *formation* (*Gestalltung*) rather than the *form*, as the primary aim of art. Dynamic figuration, which can be expressed as dynamic rule application, is based on a world of diversity, where things move freely for the sake of going, without aim or will, and without obedience. In this phase, elements are moving self evidently, in a state of primal motion. The opposite (counter concept) is what happens afterwards as the result of figuration: change, development, fixation, measurement and determination. The end, what Klee names "appearance", is only a part of what is essential. The true essential form is a synthesis of figuration and appearance. But form is momentarily and all rules are made to be broken. In art it is essential to create movement according to rules and to create deviations while bearing the rules in mind.

II.   LINE

A.   Mathematical concept

Proclus (p. 96, 21-97, 3) defines the line as "magnitude in one dimension" or "magnitude extended in one way". While the definition of the point is negative, the line introduces the first "dimension", and to this extent its definition becomes positive, while it also sustains a negation which denies the other dimensions. Aristotle (*Metaph.* 1016 b 25-27) defines the line as a magnitude "divisible in one way only" in contrast to a magnitude "divisible in two ways" which is a surface, and a magnitude "divisible in three ways" which is a solid. Aristotle also connects the line with the notion of "length", while "breadth" is connected with the surface, and "depth" with the solid. Euclid states that a line is breadthless length. Proclus gives an alternative definition of the line as " flux of a point", or the path of a point when moved. The same idea appears in Aristotle (De anima I. 4, 409): " They say that a line by its motion produces a surface, and a point by its motion a line". Proclus (p. 97, 8-13) comments that "this definition is a perfect one as showing the essence of a line. He who called it the flux of a point seems to define it from its genetic cause, and it is not every line that he sets before us but only the immaterial line. For it is this that is produced by the point which, though itself indivisible, is the cause of the existence of things divisible". Aristotle (*Metaph*. 986 a 95) notes that the Pythagoreans distinguished straight and curved lines. This distinction appears in Plato (*Republic* X. 602 c) and in Aristotle (*Anal. post.* I. 4, 73 b 19). From curved lines Plato and Aristotle separate off the "circular" as a distinct species. Aristotle recognizes broken lines forming an angle as one line. Thus, " a line if it be bend, but yet continuous, is called one" while for Heron "a broken line is a line which, when produced, does not meet itself". Aristotle also (De caelo I. 2, 268 b 17) states that "all motion in space, which we call translation is (in) straight line, a circle, or a combination of the two; for the first two are the only simple (motions)".

B.   Graphical concept

Following the Pythagorian view Kandinsky (1924, p. 57) sees the line as a derivative element. Line is as the track of the moving point that signifies the visual transition from the static to the dynamic. Movement that alters the stillness of the point requires a natural cause, which Kandinsky identifies, in the alternate or simultaneous action of one or two external forces. Here an external reason causes the transformation of an existing basic element into a new one with entirely different properties. The characteristic visual property of the line becomes the ability to evoke motion. Our eyes follow the line in its full length. The line is also divisible in infinitely many ways in opposition to the indivisible point. The line remains always a line, even in its smallest subdivision, since the notion "line" is qualitative. Thus, the line is the antithesis to the static point, as "motion" is the antithesis to "position".
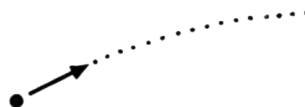

Klee

For Klee (1956, p. 105) the line is the most "authentic" basic element and the most interesting to explore because it is the most active. The point in dynamic terms is only an agent, while line is the condition of change, and the generator of endless phenomena in the domain of vision. In opposition to Kandinsky, Klee negates the idea of an external force that moves the point. The point sets itself in motion and a line comes into being. However, a restricted straight line does not produce a dynamic form. While the line in its most original, primitive condition is structure-less and develops freely the line between two points is static. The question as to whether a line is dividual, which means that can be divided freely, or individual, which means that it cannot be divided without loosing its qualitative significance, is decided by what Klee names the criterion of indefinite extension, or definite measure. The criterion that refers to the context of the computation and as a concept has deep implications in the way we compute with unstructured lines versus restricted lines. When there is indefinite extension, arbitrary divisions can be made without changing the structural style (dividual). But where an individual has definite measure, nothing can be added or subtracted without changing it into another individual (Klee 1956, p. 245)

Klee
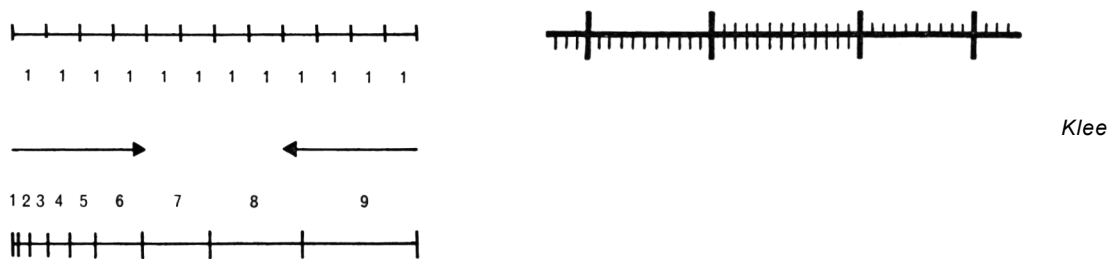
The concept corresponds to two distinct computational approaches that can be applied in different cases. The idea of definite measure corresponds to the deterministic or combinatorial mode where the use of predefined units, restricted elements, or modules that refer to a scale of distinct quantitative values, prohibits arbitrary divisions that may cause confusion in the evaluation of the components and in the way rules apply to combine them whithin the system. In that sense Klee defines these elements as "individuals" and the quantitative evaluation,or the number, determines the qualitative properties of the elements. The notion of indefinite extension corresponds to non-deterministic or unrestricted elements of arbitrary measures, where arbitrariness is guided by criteria that emerge in place, and parts are picked out by rules that act on shapes, parts of shapes, and divisions of lines. Elements and all their possible divisions or extensions are free to interact as desired, and both qualitative, and quntitative "values" emerge on the way. Klee defines these elements as "dividuals" because they can be divided or extended in infinetely many ways . Here the qulitative evaluation determines the quantitative properties of the elements. Each of two modes produce corresponding structures that Klee names individual and dividual structures. However, arrangements of elements can be approached as dividual or individual, depending on how we choose to treat them. Arrangements that include determined number of parts can be seen as individuals, while if the number of parts is undetermined can be seen as dividuals.

Examining the concept of dividual and individual in relation to shape algebras, in U_0 algebras points and shapes with points are individuals , whereas algebras that include lines and planes can support both dividual and individual structures. The combinatorial approach can be suitable when standardization of components and forms is preferable or where preanalysis is possible, whereas the non-deterministic approach avoids the need for foreknowledge, leaves more freedom to innovation, and appears more open-ended. As an extension of the above ideas the two approaches can be combined to produce composite forms in which individual structures produced in the deterministic way combine with dividual structures produced in the non-deterministically. An example of composite structure, is the combination of the structural grid of a building (parti), which is individual, with the actual plan of the building which is usually dividual. For Kandinsky and Klee the formal elements of graphic art are points, and linear, planar, and spatial energies. In the general case basic elements are used in formal composition as distinct qualities. Linear, and planar forms can be constructed from shapes that are arrangements of lines and planes. Planes, and lines, weighted or not, are combined through application of compositional rules to produce forms. Also operations like addition, subtraction, and multiplication and the set of transformations are explicitly introduced by Klee (1956, p. 159) as an essential part of a computational machinery of dynamic figuration.

Addition

Klee
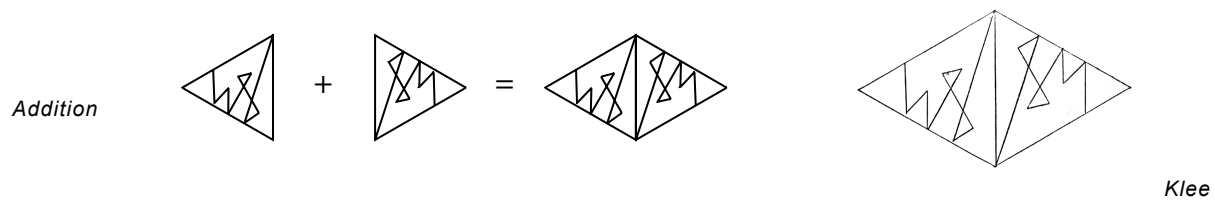
The process of composition becomes equally important with form. In the general case when a linear form is combined with another linear form according to a rule, the linear parts are considered to have "active" character. In a linear shape that lies on a finite plane the linear part takes on a decidedly active character and the plane a passive character. Klee suggests that the active part, the line, can accomplish two things: it may divide a planar form into two parts or it may give rise to a displacement of the parts (Klee 1956, p. 7).

Klee
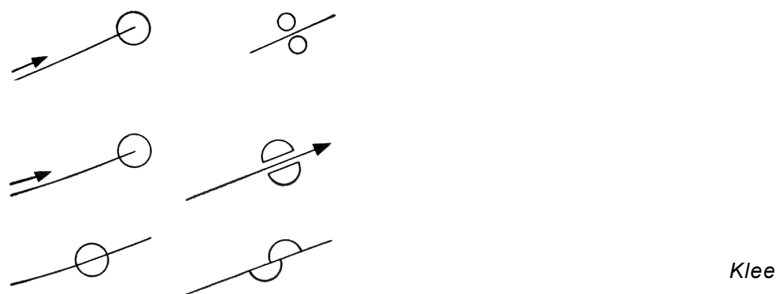
In shape grammars, a line is a distinct quality and is considered always a line. The line is bounded by definite and distinct endpoints, and has finite nonzero length. Linear forms made up of linear shapes can be manipulated on the plane, or in other dimensions according to rules. In the case of two-dimensional space, a shape is a finite arrangement of lines on the plane. A shape can contain occurrences of straight or curved lines that are connected or disconnected. Rules that refer to relations between shapes apply to generate new arrangements. The linear parts can be embedded in one another can be separated, can be added, subtracted, or transformed. (rotated, reflected, translated or scaled).

Klee

C.   Dictionary

Klee (1924, p. 23) defines the three formal means that apply to all basic elements as length, weight, and color. Length is quantitative and can be equated with measure, weight is medial can be equated with texture, and color is qualitative. Quantitative properties like length, angles, length of radius, and focal distance, are subject to measurement. Weight is a "relation" and includes tone value, which can be expressed as degree of shading between black and white, and as texture. Finally the color can be neither weighed nor measured. According to Klee neither with scales nor with ruler can any difference be detected between two surfaces, one pure yellow and the other pure red, of similar area and similar brilliance. Basic lines are considered the straight and the curved. Definitions of elements like the vertical, the horizontal, and the diagonal lines, are relative and refer to a specific graphic context determined by a coordinate system, while definitions like that of the angled or the curved lines, are independent of context. In shape grammars elements are defined only in relation to a coordinate system, while the graphic context of the dimension in which we manipulate the element is considered infinite.

1.   Straight Line

Kandinsky (1924, p. 57) illustrates the generative cause of a line as action of one external force to a point. In their abstract definition of straight lines Kandinsky and Klee see the line having infinite length, therefore consideration of endpoints seems unnecessary. However, this need soon appears reasonable to Klee who observes that any line can be the result of tension between two distinct points. In shape grammars a line is always bounded by definite and distinct endpoints, and has finite nonzero length. Moreover, a line can be transformed to any other line when is embedded on it and its endpoints are moved. This happens naturally if we draw one line on another and for Kandinsky and Klee there is no need to define this relation. In shape grammars lines can have common linear parts or can be discrete, whereas for Kandinsky and Klee linear forms can also touch in points. In both theories the position of lines on the plane and their relation with other lines is determined by the position of their endpoints. Kandinsky (1924, p. 57) distinguishes lines in *typical* and *atypical*.

1.1   Typical straight lines

Typical straight lines are the horizontal, the vertical, and the diagonal because all three can be perceived, and executed without measurements. Kandinsky sees the horizontal as a flat supportive base while in complete contrast the vertical, has height. For Klee this pair of lines refers directly to the symbolic scale of the coordinate system, where the horizontal is dependent on the vertical. The diagonal diverges equally from both.
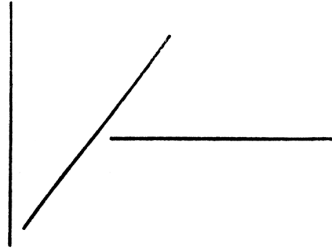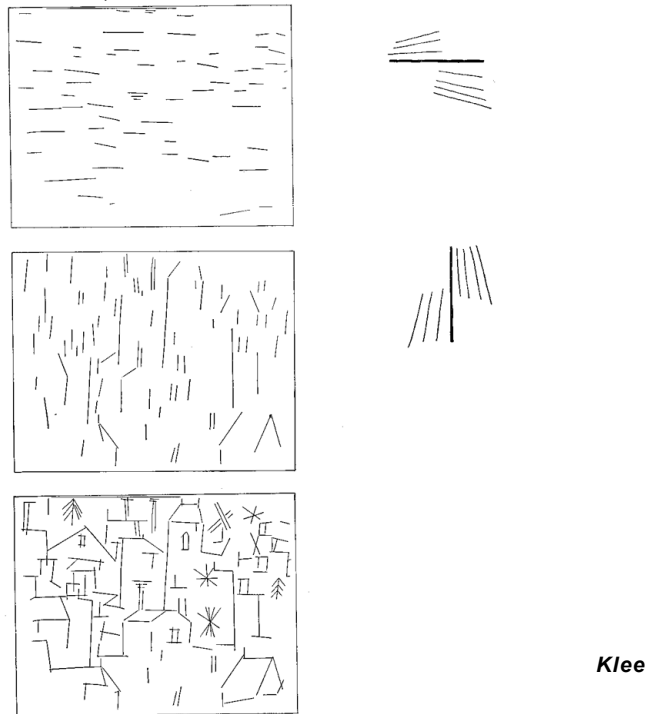

Kandinsky

1.2   Atypical straight lines

Kandinsky names *atypical* all straight lines that are not horizontal, vertical, or diagonal (1924, p. 63), because their relation to the coordinate system cannot be determined visually. However, in a given finite context, which is determined by a finite plane, Kandinsky implies that we can refer to the center and characterize these lines as *centric* or *accentric*.
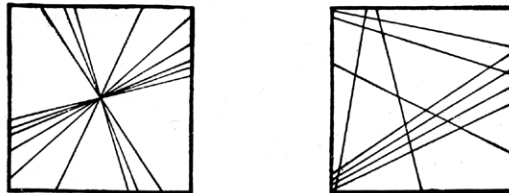
Kandinsky

In shape algebras elements are considered finite but the space in which we manipulate these elements is infinite. Still the above organization of lines in relation to the center of a finite graphic context, or in relation to multiple "centers", could be constructively useful in painting and in urban planing or architecture, where arrangements of blocks and roads need to be organized in relation to particular centers. The above organization of lines could be attainable by defining the limits of the dimension in which we manipulate the element, in this case the plane. Then, the intuitively suitable notion of finite graphic context needs possibly to be examined as an optional feature of a U_{ij} algebra in order to accommodate the standard practical demand for rule application in the finite surface of a canvas or a specific site in the case of architecture. One example of constructive application of the above ideas in painting is illustrated by Klee (1956, p. 190). In these arrangements of straight, vertical, horizontal, diagonal, centric and acentric lines, a predetermined scale of convergence to the vertical and the horizontal and multiple centers are applied. The visual effect is the creation of pictorial depth.

Klee

2.   Angular and curved lines

Kandinsky and Klee categorize angular lines, with curved lines as the result of a more complex movement of a point by two external forces. Moreover the angular line is considered as one line. The action of two forces in the realm of the line can take place in two ways:

1. Point $\rightarrow$ Alternate action of two force $\rightarrow$ Angular line
2. Point $\rightarrow$ Simultaneous action of two forces $\rightarrow$ Curved line
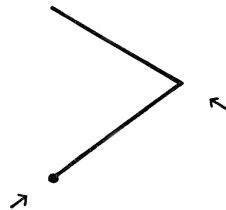
Kandinsky

2.1   Angular line

The notion of an external force that acts on one element, the moving point, and changes its straight course, is a mechanical way that Kandinsky employs to describe how a given context interacts with the straight line and changes its form. Angular lines are also described in Klee's illustration of a sequence of straight restricted lines. The angular lines in computational terms can be seen as shapes that contain a finite number of straight maximal linear elements and are manipulated in a U_{11}, U_{12}, and U_{13} algebras, where the part relation holds if the elements have common linear parts. The U_{11} algebra contains one straight line that is the special case of an angle of 180°. For the rest of the algebras maximal linear parts combine on the plane and the three-dimensional space in a way that every two share one common endpoint and in the general case do not coincide. An extension of the concept of angular lines is also the notion of angular planes, or solids that are manipulated in space in U_{23} and U_{33} algebras. While straight lines can

be described as length, angular lines depend on two characteristics: their angles and the lengths of their straight linear segments. Many-angled lines can also derive by addition of pairs of simple angular lines.
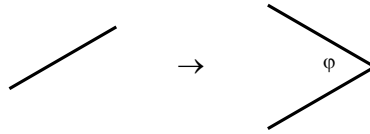

2.1.1   Angles and lengths

The angle is the simplest case of angular line.For Klee and Kandinsky the notion "angle" is qualitative and belongs entirely to the domain of vision, without further numerical extensions. In shape computational terms the simplest angular line contains only two maximal linear seg- ments that share one endpoint, and in the general case do not coincide. Angles are character- ized by Kandinsky (1924, p. 69) according to visual criteria as *typical* or *atypical*.Typical are the angles of 45°, 90°, and 135° because their characteristically exact angle-sizes can be perceived and executed intuitively without measurements, while all other angles are characterized simply as *atypical*.

	Angle	*Quality*	
α.	Acute – 45 ·	sharp	
β.	Right – 90 ·	controlled	(*Typical*)
γ.	Obtuse – 135 ·	passive	
δ.	Free		(*Atypical*)

In shape computation a parameter φ can be introduced in a shape rule to express the value of the angle and to construct different angles. The shape rule takes the initial line on the left side and returns an angle with value φ on the right side. The further difference between the simple angular lines consists in the lengths of their individual linear sections, which greatly modifies the basic character of these forms. In the sense that Kandinsky describes thegenerative cause of angles, as action of one external force to a staight line, we may parameterize further limita- tions in the values of angles and lengths according to external conditions and translate the interaction of an external cause with straight line into specific angular forms.
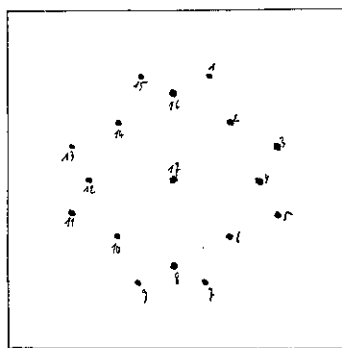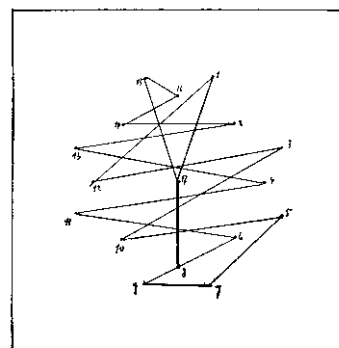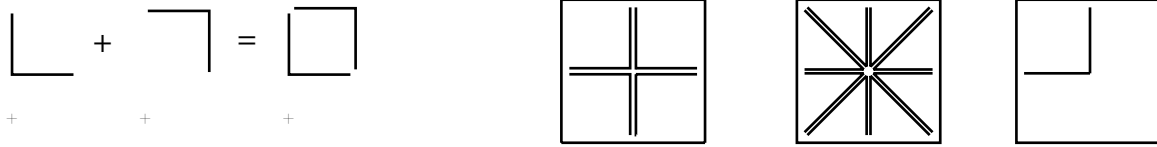

Klee

Typical angular forms have distinct ways to tile the plane. Accordingly, four right angles tile the plane and can form a cross, or a square, eight acute angles tile the plane, while, the obtuse is not capable of tiling the plane exactly but two obtuse angles added together leave an exact portion of 90° unconquered.

2.1.2   Compositional parameters of angular lines

Kandinsky (1924, p.70) introduces a concept that is related to architecture. Although an angular line is described as lengths and angles, its potential in formal composition depends on three parameters. These parameters can be seen as one composite parameter, they can be used singly, or in pairs.

α.   The lengths of their straight parts.
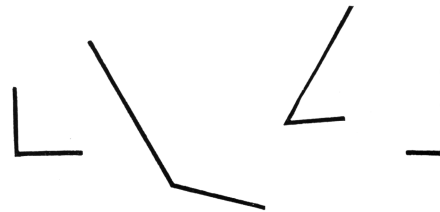
β.   The inclination of their angles (more or less acute).
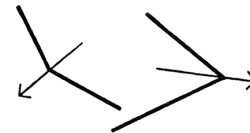
Kandinsky

γ.   The conquest of the plane.
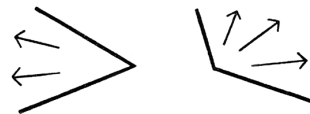
Besides the lengths of the straight segments and the angle, the third parameter relates the angular line to its context. While the two first can be expressed arithmetically, the third belongs entirely to the domain of perception and it is relative to the context of the composition, so that numerical values do not apply. Here Kandinsky introduces a significant concept for the qualitative difference between straight linear and angled elements. While straight linear elements, simply "divide" in a more or less neutral way, angles have the tension to "conquest" in distinct ways, something that we experience stronger in architectural space. An acute angle made out of planes conquests the space in a distinctively different way from an obtuse, or a right angle etc. The same concept is implied by Alberti (1955, p. 1-2) who defines "design a firm and graceful preordering of the *lines* and *angles*, conceived in the mind, and contrived by an ingenious artist" thus establishing angles as a distinct quality of architectural form, although angles are made of linear parts.

2.2   Curved line

Kandinsky (1924, p. 79) by applying the concept of external force generates the curved line from simultaneous action of two forces on the point where the one force acts steadily and continually and exceeds the other. This type of line exhibits the tendency to close itself. The simple curved line and the straight line have different character. If the straight line expresses the possibility for endless extension, the curved line has "elastic" character. Moreover, Kandinsky according to the Pythagorean approach notes that the straight line and the curved line constitute the primary contrasting pair of lines, because while the angled line includes straight segments, in the curved line we notice complete absence of the quality "straight line". In a computational environment forces that are not elements of design, but that act on basic elements, could be introduced as another means of shape transformation or as away to inter-pret shapes.

Kandinsky
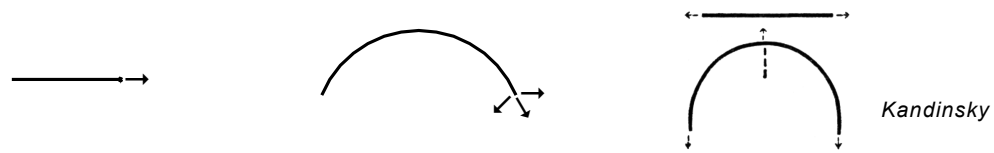
Klee (1956, p. 123) sees the curved line as the antithesis of the straight line, a form that has something restful and harmonious. The non-geometric curve is an expressive suggestion of a restful walk without purpose, and in musical terms suggests a folk-song rather than a more elaborate form. However, the curve can also appear in restricted form, and can be generated in relation to two given points.

Klee
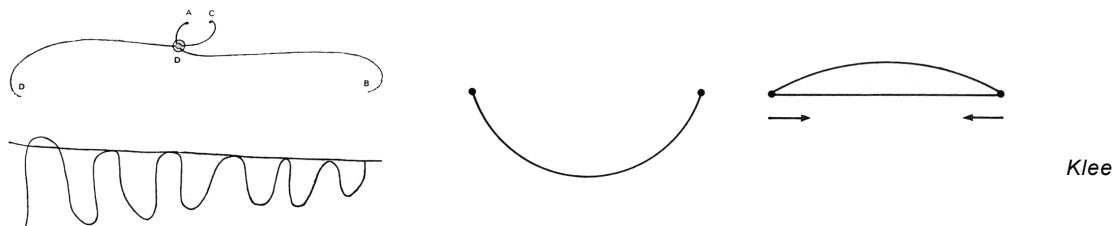
Curves are lines that can be manipulated in a U_{11}, U_{12}, U_{13} algebras like other lines. Geometrical curves are easier in description with numbers, where free non-geometrical curves are more complicated forms. The embedding relation and the transformations need to be extended to calculate with free curves. However, complex forms of curves are usually combinations of simpler curved segments. Kandinsky (1924, p. 85) offers computational rules for the construc-tion of complex wave-like lines that cover all the forms of curves. Accordingly a complex curved or wave-like line consists of geometric parts of a circle, free parts, and various combinations of these (equal or unequal radius, horizontal or vertical course, and alternating tensions). Shape rules can be introduced in U_{12} algebra, to illustrate this idea,

Curve geometric wave-like

D.   Composition

Kandinsky (1924, p. 74) observes that in formal composition the components produce a sum, but there is not always direct correspondence of the character of the sum with the characteristics of the components. The same happens in chemistry, where the sum divided into its components in many cases fails to be restored in the combination of the components. Therefore, the equality sign is not used but rather, an arrow ↔ which points to relationships. Kandinsky suggests that the quest of his research is "the relationships between the elements". The sum, or the form, simply supplies the missing factor necessary to balance. The components would be derived from the sum and vice versa. The same idea is expressed in shape grammars with the concept of rule application, or spatial relation, with a "left" and a "right" part. This corresponds precisely to the notion of a relationship, where components and solid descriptions are not preserved since may restrict elements to interact freely. Multiple descriptions can emerge from rule applications on elements and the components may vary. The role of rule application in this case is also highly emphasized by Kandisnky (1924, p. 92) who states that only the action of a rule on an element brings life to that element, and allows to its basic properties to be expressed. Accordingly, he defines composition as an exact law-abiding organization. The composition with lines is the clearest and simplest example of this creative process which always takes place according to rules, and therefore allows and requires rule application.

1.   Lines, angles and shapes

Simple straight or angular lines produce shapes. We need at least three lines or angles to compose the simplest shape, and with it the primary plane made out of straight lines, whereas, one curved line can produce the primary plane of a circle. Thus, the triangle and the circle become for Kandinsky the contrasting pair of shapes. A further correspondence between lines, planes, can be established. The qualitative characteristics of angles are used to point to planar shapes. The basic planar forms are introduced as products of angular lines. The square, the triangle and the circle become the typical forms of the plane. Right angles, with balanced qualities construct the square. Sharp acute angles construct the equilateral triangle. The passive obtuse angles are distantly related to a form without angles. They constitute the third primary shape of the plane, the circle.

Angles   *Forms*
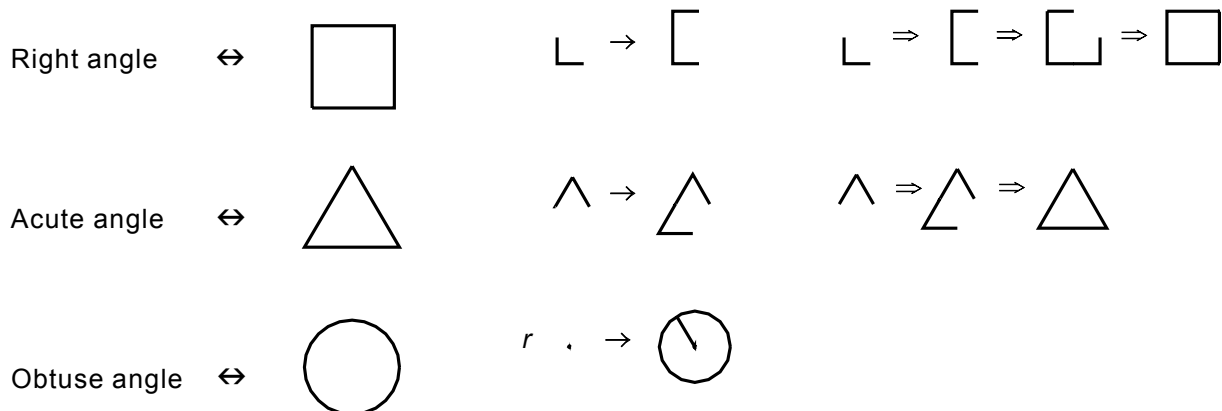
2.   Complexes of lines

A further extension of the concept of subordination of elements according to rules is possible for lines. Kandinsky (1924, p. 93) attempts to expose the constructive power of two principles, the principle of the parallel and the principle of the contrast. The visual result is more complex forms and articulation of different rhythms, depending on the quantitative or qualitative nature of the repetition and of the linear elements. The two compositional rules of parallel and contrast can also be expressed as rules in a U_{12} or W_{12} algebra

Repetition of straight line with alteration of weights or intervals:

Repetition, or opposed repetition of angular lines:
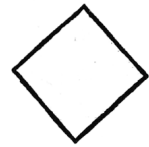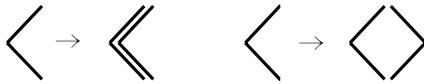
Repetition, or opposed repetition of curved line:
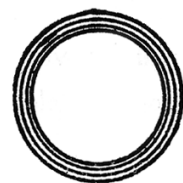
Contrasting repetition of curved line:
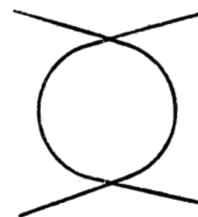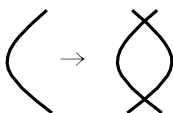
Kandinsky

3.   Linear, medial, and planar shapes

In U_{ij} shape algebras boundaries of elements are defined in an algebra U_{i-1j}. This expresses the fact that points bound lines, lines bound planes, and planes bound solids. Examining the inter-action of lines on the plane, Klee and Kandinsky refer to two distinct cases. First, finite arrange-ments of lines that have at least one of their endpoints not common or lying in between the endpoints of another line are considered linear active elements. Free straight lines, or angles that share one common end point or arrangements of lines about a common center are included here.

Klee
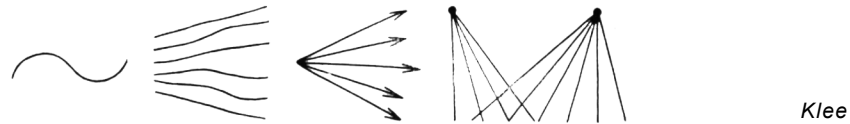
Second finite arrangements of lines that compose closed linear shapes are considered medial. This follows the Euclidean definition of shapes where a figure is considered shape if and only if is contained by closed boundary or boundaries. Klee (1956, p. 111) observes that lines by circumscribing a shape bound an area of the plane and cause planar effect. If we see a shape as an action of the hand the line remains always a line. But when it is completed the linear impression inevitably changes to a planar impression and the linear character disappears. For the same reason Klee characterizes closed shapes hybrids. For Klee and Kandinsky shapes can be seen as combinations of lines or as boundaries of planes depending on how we want to look at them, and therefore they are considered medial. All free and geometrical shapes are included in this category,

Klee

In the case of weighted or colored planes when the boundary-lines are not visible as distinct elements lines are considered to be "inactive" and the planar character of shapes prevails.

Klee

Then, arrangements of straight lines and closed curved lines can be visually associated to planes. Kandinsky observes that geometry does not make this distinction which is exceedingly important for graphic representation and mentions the qualitative difference between the circle and the spiral. Although both are products of the curved line, the spiral can be perceived only as a line, where the circle may also be seen as a plane.

Klee

This ambiguity of shapes may have implications in the way they combine. Klee distinguishes three modes of interaction: shapes may be discrete, may lie side by side, or overlap. The relation between their parts may be, no contact, or contact in point, in line, in plane, or in space. Accordingly, parts of shapes may be apart, touching (one-dimensional contact) or inter-penetrating (two-dimensional contact).


Klee
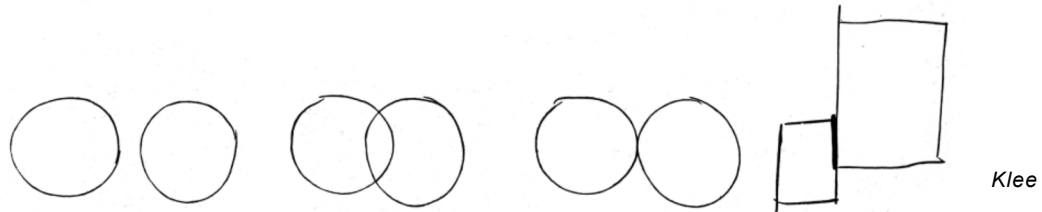
A shape in a U_{12} algebra is an arrangement of lines that lie on the plane. Shapes may be discrete, or may have common linear parts. Shapes with lines that share points are considered discrete since they don't have common linear parts. More generally, in shape grammars shapes have common parts only if the parts belong in the same algebra with the shapes. They do not have common parts otherwise. Thus, in the above example in a U_{12} algebra only the two rectangles are considered to have common parts, whereas Klee would characterize, the first case discrete, the second touching in a point and the third, interpenetrating. In the fourth arrangement the rectangular shapes both in a U_{12} algebra and in Klee's approach have a common linear part, in the case we see them as an arrangement of lines. If we see them as an arrangement of planes, in a U_{22} algebra, the shapes do not have common planar parts. The above relations are illustrated in the following table,
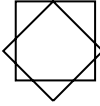

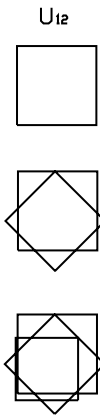|				
Klee	$\varnothing$	(lens outline)	•	|
U_{12}	$\varnothing$	$\varnothing$	$\varnothing$	|
U_{22}	$\varnothing$	(grey lens)	$\varnothing$	$\varnothing$
$U_{22}\ U_{12}$	$\varnothing$	(grey lens)	$\varnothing$	|

Klee's characterization of shapes as medial gives him enough flexibility to see them and manipulate them as arrangements of lines or as boundaries of planes. The difference with U_{12} algebra appears stronger in the second example, where Klee sees a common linear part between the two shapes. This intuition to pick this part as common between the two shapes, is irrelevant from our knoweledge of the dimension in which they belong and expresses the continuous flux between dimensions. A compound $U_{22}\ U_{12}$ algebra, which contains planes and

lines manipulated on the plane would produce almost the same graphic environment that Klee characterizes "medial", since it contains both the planar shapes and their linear boundaries. However, the notion of a more inclusive medial graphic environment, where shapes made out of lines can optionally be approached as planes, or they may have common parts without necessarily having common linear parts could be more appropriate. Still the introduction of a specific shape rule could possibly capture more expressively Klee's approach, just for the case. The rule in a U_{12} algebra would pick any linear part we need from two shapes without altering the algebra.

U_{12} U_{12}
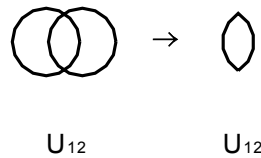
To incorporate better the option to perceive shapes in different dimension and to be able to calculate with them in a more systematic way, complex cases of similar visual transitions would probably demand more thoughtful action. An alternative would be to retain both descriptions of the element or the elements that "change" or transfer to another dimension, and to proceed in parallel computation, in different algebras or combinations of algebras. In this case one can always refer to the step that the transition occurred or to any of the indermidiate steps and proceed from there. This resembles the parallel exploration of ideas in architectural composition, where shifts in the way one sees and manipulates shapes occur naturally. In each of these stepsof transition the use of tracing paper leaves intact or informs the previous stages of the composition. In the following example a rule places squares into squares, and provides a basic form that can be explored, or decomposed in various ways. In the case that one is interested to explore potential solutions for the plan of a house,one would go through series of parallel computations. The one that is included here transfers linear parts of the arrangement, to planar parts that correspond to specific spaces.

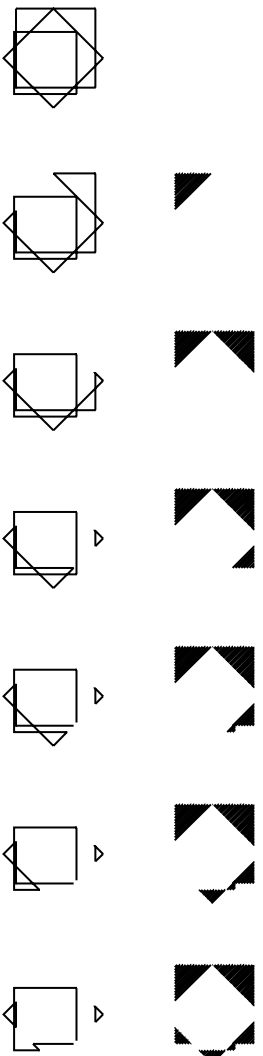|		
rule 1:	U₁₂ U₁₂	
rule 2:	U₁₂ U₁₂ U₂₂ U₁₂ U₁₂ U₂₂	
rule 3:	U₁₂ U₁₂U₂₂ U₁₂ U₁₂U₂₂	

Derivation A: using only rule 1

(import shape from Derivation A)
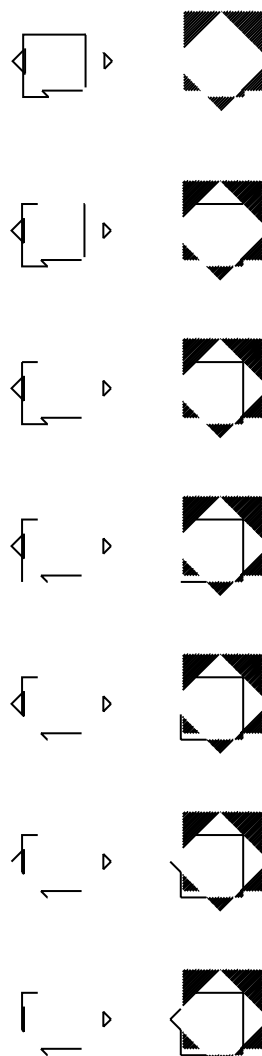
U₁₂   U₁₂ U₂₂

Derivation B: using only rule 2

(import shape from Derivation B)

U₁₂   U₁₂ U₂₂

Derivation C: using only rule 3

4.   Divisions of shapes

Can we define "elementary" divisions in arrangements of straight lines? In the following example a horizontal and a vertical line are placed centrally on the square plane. This arrangement is considered by Kandinsky (1924, p. 66) an extension of the concept of the centrally placed point in the domain of lines.
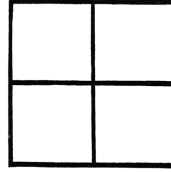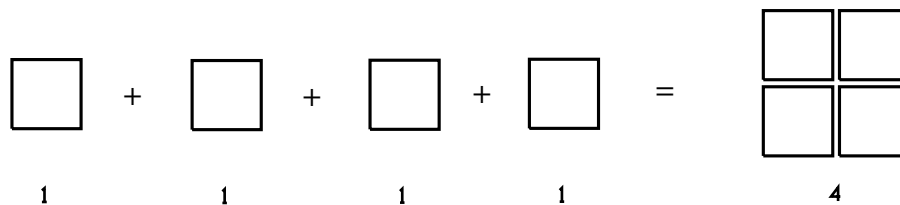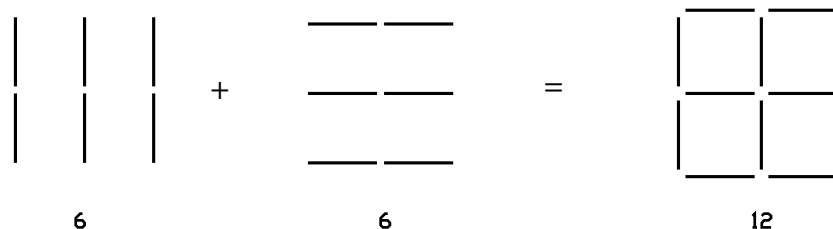

Kandinsky

The issue here is to define the "elementary parts". Kandisky manipulates the plane of the finite graphic context as part of the composition and sees the arrangement in various ways. Visual transitions from from plane to linear shapes occur spontaneously.The initial arrangement is defined here in a combination of U_{22} and U_{12} algebras, which contains a square plane and two lines, manipulated on the plane. A parallel computation takes place only with lines on the plane in a U_{12} algebra. First we see this shape as the result of interaction between 1 plane and 2 lines. This gives a description that contains three elements.


1   2   3
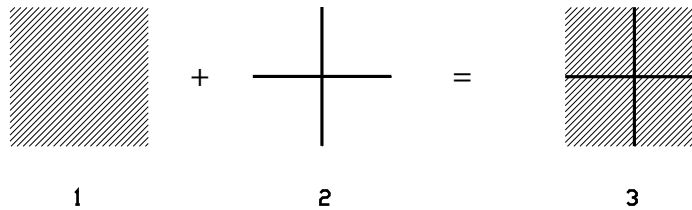
However, Kandinsky notes it is equally permissible to see the arrangement as four equal squares made out of lines, in a U_{12} algebra. The three previous elements can be seen as four new elements of entirely different nature.

1   1   1   1   4

Kandinsky observes that further possible elementary divisions of this basic schema exist: six horizontal and six vertical lines could be seen as the elementary parts of this arrangement, again in a U_{12} algebra. The sum consists of 6 + 6 =12 lines.

6   6   12

Or, we can alternatively see the arrangement as 4 planar elements + 2 linear elements = 6.

| 4 | 2 | 6 |
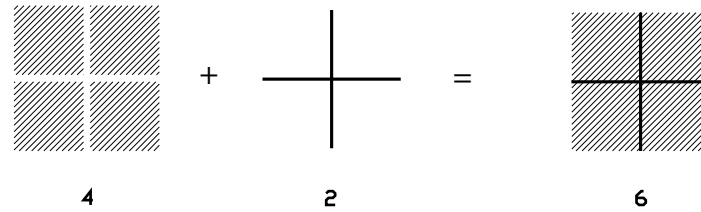
The results of Kandisky's attempt to define "elementary" divisions of the arrangement is illustrated in the following table, in parallel with a reference to the shape algebras that the parts belong.
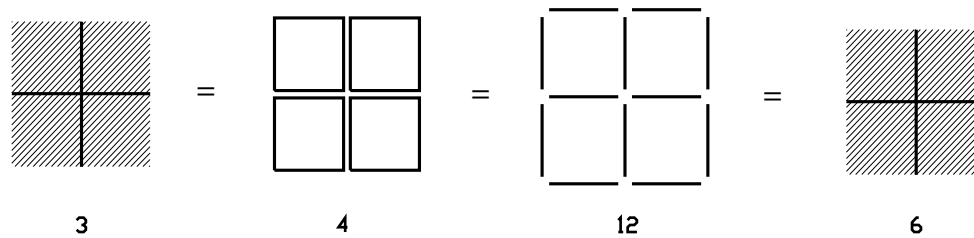
| 3 | 4 | 12 | 6 |

| U_{22} | 1 plane | - | - | 4 planes |
| U_{12} | 2 lines | 4 linear squares | 12 lines | 2 lines |

Kandinsky (1924, p. 66) ends with the conclusion that the notion of "elementary" combinations, or divisions is relative. Relativity is part of the nature of shapes. Any choice between combinations of elements remains open and can describe the same arrangement equally well. The characterization "elementary" is clearly inadequate and imprecise, since it is not easy to fix a limit for the complexity of shapes, or to reduce this complexity into elementary parts. However, Kandisnky observes that the analysis of shapes through observations like the above offers the only means of getting to the bottom of pictorial things, and serves the ends of composition.

A further conclusion that we can draw from Kandinsky's observations is that the ambiguity of graphic forms arrives from both ends, it is related to transitions of shape and dimension. Basic elements may compose finite arrangements that we call shapes in a specific algebra in infinitely many ways and shapes may change their basic components,in the shape algebra at any demand. In that sense infinite compositions and decompositions of the same arrangement are possible. Moreover, the dimension of these parts may be subject of continuous change. Parts of shapes may turn into elements of a different algebra. The expressive power of basic elements is based on their ability to interact in ways that are not predetermined in any sense and depend exclusively on the eye of the beholder. This ambiguity of form and of dimension makes basic elements abstract and inclusive, while at the same time they are concrete and visual, ready to support any possible interpretation. The multiplicity in the way we see and treat forms in architectural design is highly dependent on these visual transitions where the dimensions and the parts of graphic elements constantly change. In shape grammar theory the organization of basic

elements in distinct algebras and the possibility to transfer shapes from one algebra to another, to calculate simultaneously in different algebras, and to combine them, serves primarily the demand for dimensional transitions, while, the shape rules, the identities and the part relation that allow parts to be composed and decomposed in infinite many ways satisfies the normal expectation that forms do not have predetermined parts. However, visual transitions of dimension that occur spontaneoulsy in computations and cause phenomena of perceptual flux, need to be treated as a significant factor of ambiguity and creativity and need to be examined and supported computationally in further detail from shape grammar theory.

### E. Graphic properties

1.   Boundaries and texture

Kandinsky (1924, p. 91) observes that the outer boundaries of the line can be considered as two independent lines that are not necessarily straight or continuously parallel. Then, the edges of the line can be seen as having a shape. These outer edges may undertake qualitative and expressive significance. They can be smooth, jagged, torn, rounded etc. Since line has length the combinations are more complex than those in the case of points and each of the two sides of the line can have a special character. However, if we remain in the domain of lines, and we do not consider a specific line segment as a plane, the form of these boundaries can be treated as part of the line texture in a W algebra (W_{11}, W_{12}, W_{13}). The importance of weights as a means of graphic expression is emphasized by Klee. The slightest change in weights of lines can alter the perceptual value of the elements. For Klee weight is "medial" parameter is in between length and color, where length is characterized as "measure" and color as "quality".
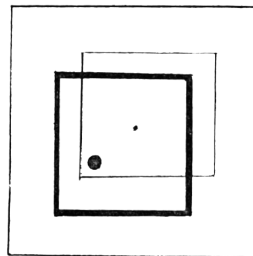

Klee

2.   Emphasis

In this case that the shape of the line- boundaries develops to a significant visual parameter for the quality of the line, the weighted line can be approached as having specific atributes of form. A special case of the above is when a specific visual effect of increase or decrease of strength is desired. Kandinsky (1924, p. 89) names the effect emphasis. Here the transition from a simple line to a weighted line occurs as a more radical shift and weight is used as a computational means to enable the line to attain the shape necessary at the moment.

U$_{12}$ **W$_{12}$**
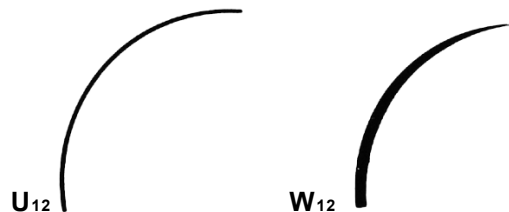
Kandinsky

3.   Transition to plane

Any short weighted line often bears a relation to the plane. Kandinsky (1924, p. 90) observes that as in the case of the point the boundaries of this transition cannot be determined analytically but are indefinite and mobile. The visual outcome depends upon proportions, relations with the other forms of the plane, and more importantly perception. However, this visual transition from line to plane becomes a potent source of expression and of unexpected consequences in computations with basic elements. Here one is dependent upon visual intuition and personal will, since a predetermined distinction between line and plane is impossible. In shape computational terms a line on the plane in a U_{12} algebra, or a weigthed line in a W_{12} algebra needs often to be approached and manipulated as a small plane. Rule application that transfers the element in a U_{22} or W_2 algebra which contains planes or weighted planes, can serve this demand.

$$\underline{\hspace{3cm}} \quad \rightarrow \quad \blacksquare$$

U_{12} U_{22}
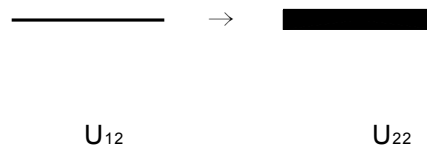
In the more general case, any line, or element that belongs to a specific algebra can change dimension. The constructive interest of this transition appears in the computations that follow, since designers manipulate in a different way and for different reasons, a line from a plane, or a plane from a point. As Klee (1956, p. 78) points out the work of art is first of all continuous genesis. That means is never experienced purely as a *result.*The given is a whole consisting of various unknown parts, and the problem is to find them. This implies also fluidity of dimension. What the designer does is temporal: the liberation of the elements, their arrangement in subsidiary groups, simultaneous destruction and construction towards the whole. After a while someone stops and goes away or if something strikes him returns and may approach things in an entirely different way. Abstract formal elements are put together to make concrete or abstract things. In the end the form is achieved by a ceaseless action. The computational interest appears in the manipulation of elements in terms of parts and in terms of dimension. Rule application is equal with a decision to start looking, and more importantly manipulating, the same element in a different way with the only aim to explore the implications for the composition. The possibilities are infinitely many, and as most designers experience decisions in this procedure cannot be definite beforehand . Questions remain to be answered, in relation to transitions of dimension and the possibility for a more general treatment of occurrences of dimensional flux computationally.

III.   PLANE

A.   Mathematical concept

Proclus (Proclus, p. 117, 8) defines the plane as "a surface such that a straight line fits on all parts of it" and Aristotle (*Metaph*. 1020 a 12) as a magnitude extended two ways, or "divisible in two ways". Aristotle (*De anima* I. 4, 409 a 4) mentions the common remark that "a line by its motion produces a surface" or that is "the extremity of a solid" and Euclid (*Elements*, I. 7.) states that "a plane is a surface, which lies evenly with the straight lines on itself". In the modern approach Bolyai and Lobachewsky (*Elemente der absoluten Geometrie*, 1876) evolved the plane as "the locus of all points equidistant from two fixed points in space". Leibniz (*Uber Euklidische Geometrie*, 1888) defines plane as "the surface which divides space into two congruent parts" with the further condition that the two congruent surfaces could be "slid along each other without the surfaces ceasing to coincide".

B.   Graphical concept

Kandinsky and Klee refer to the plane in two ways. Plane is a planar shape or a form with closed boundaries. It is also the basic graphic surface. The material plane of the graphic sur-face becomes the finite graphic context that has closed linear boundaries and is approached as a shape of the composition. What is of importance, in this case, is the interaction between the plane and the other basic elements that appear on it. The relativity of value of all forms, and the ambiguity of their dimension can be exposed to some degree in comparison with other forms, and in comparison with this basic finite graphic context. This context and its linear parts may define the relative value of an isolated form as well as of the whole sum of forms.

1.   Planar forms

Klee (1956, p.112) generates the plane from the moving line. The trace of a constant motion of a straight line on the plane can be perceived as a plane with rectangular or square shape. Also, linear rotation about a point generates a planar circle. The line becomes invisible as element, or passive according to Klee (1956, p.112) while the plane becomes visible or active.


The above plane can be expressed computationally as a weighted or colored plane in a W_{22} algebra or as a simple plane in a U_{22} algebra. But what Klee sees as plane form, on a plane surface, is what emerges from the coloring of the area of a closed shape. He names it, transi-tion from the medial shape (or inactive plane) to the active plane. Then, each active planar element corresponds to a medial shape, which is equal with the shape of its boundaries. What makes the plane visually active is the "activation" of its full area, and the parallel visual "disactivation" of its boundaries as distinct elements. A compound U_{12}, U_{22} algebra, which

contains both the linear boundaries and their enclosed planar area, would support this effect. In the case of the passive plane the planar part that is contained in the U_{22} algebra, remains inactive, while the linear part in the U_{12} algebra is visible or active. Then, the activation of the weighted or colored planar part could be equated with what Klee names visual transition form the passive to the active plane.

$$U_{12}\ U_{22} \qquad U_{12}\ U_{22}$$
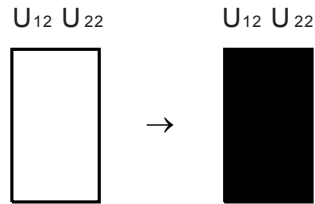
In another derivation of the plane from the line Kandinsky (1924, p.60) organizes straight lines about a common meeting point, according to the rule
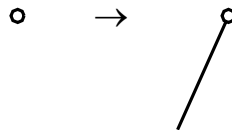
After several repetitions the shape becomes denser and the intersections form a more compact center, in which the point seems to grow. As in the case of the point, which becomes plane Kandinsky attempts to express computationally common intuitions about basic elements, and their transitions from one dimension to another.
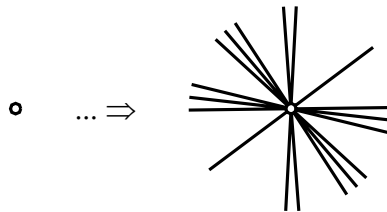
The point can be seen as a center about which lines move and finally, flow, into one another to construct a plane. This derivation of the plane is similar with the one we see in Klee and has Aristotelian origins.
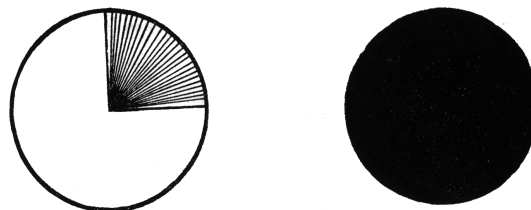
Kandinsky

Kandinsky introduces the notion of the plane as emergent shape. The plane can emerge from accumulation of lines on the graphic plane. It is stated that this is a characteristic property of the line.

The abstract plane in its general form is a closed line. This concept is illustrated in both Klee and Kandinsky in accordance to what Euclid would have called plane figure or shape: "that which is contained by any type of linear boundaries". Accordingly, Kandinsky organizes planes from the form of their linear boundaries. He distinguishes them as simple geometrical planes and their derivatives, displaced geometrical planes, and free planes. Simple geometrical planes are considered only three planar shapes, the triangle, the square and the circle. With the derivatives form the following group of six shapes.

Triangle ↔ Trapezium   Square ↔ Parallelogram   Circle ↔ Ellipsis

Displaced geometrical planes are all planar shapes contained by straight or geometrical curved lines and not classified as simple geometrical planes or their derivatives. Here are included all possible parts of simple geometrical planes, and all emergent planar shapes that have no free lines as boundaries.
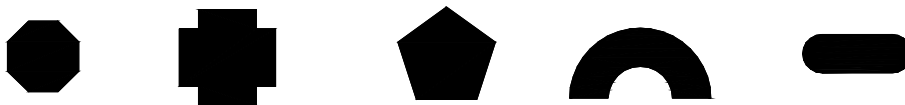
Free planes are all planar shapes contained by closed free lines.

This organazation of planar shapes is part of the analytic approach of the properties of graphic elements and the establishment of possible relationships. Here the distinct attributes of straight and free lines, or acute and right angles become the criteria for the classification of shapes. However, for Klee (1956, p. 60) and Kandinsky (1924, p. 74) exactness should never be one sided. Graphic forms cannot be conceived apart from rules that give them life. The indispensable imaginary part is incorporated in rule application. When Klee declares that "what we are after is not form but function" expresses this concept. The work, he explains "is related to the rules inherent in it. It grows in its own way, on the basis of these rules. But the rules are not *a priori*". Klee characterizes the work as projection and as phenomenon "for ever starting" and "for ever limited". It does match the infiniteness of the rules in this: even in its limited sphere the reckoning does not come out even. Thus, he suggests, "consider the actual with benevolence but do not define today, define backwards and forwards, spatial and many sided. A defined today is over and done for".

1.1   Interaction of planes

Colored or weighted planes that are manipulated on the plane in a W_{22} algebra possibly overlap. In this case their common planar part, which is the product of what Klee (1956, p. 117) names two-dimensional contact on the plane, or planar interpenetration, becomes a territory that demands treatment according to constructive and qualitative relations. Klee notes that when planar shapes interact this way their common parts can intermesh, or one absorbs the other. The following example illustrates schematically the basic possibilities for interaction between two colored planes in W_{22} algebra.
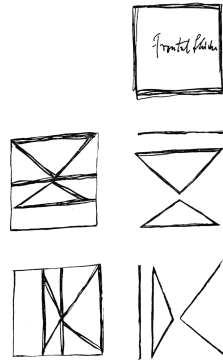

Klee
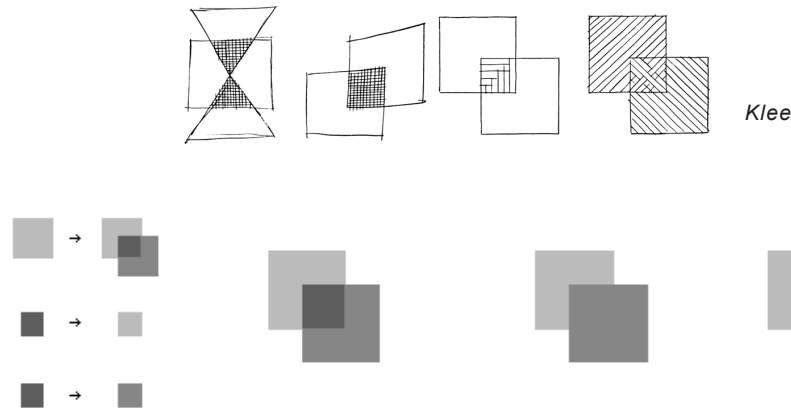
In the first case their common part becomes of a third distinct quality, while in the second and third the quality of one or the other plane becomes quality of their common part. When numerous planar shapes overlap the treatment of their common parts can be more consequential for the composition. The overlapping can be approached in a variety of ways especially when different colors and textures interact. In architectural representation, the interaction of planes except from visual results, like the creation of graphic depth, can also signify the interaction between architectural elements. In handling this qualitative relationship between planes, at least two alternatives appear computationally plausible. As is examined in Knight 1989, we can create a graphic "scale", which is a set of qualitative rules that establishes the "ranking" of planes and defines the qualities of their common parts beforehand, or, we can treat them by defining the preferable result with a shape rule each time the overlapping occurs.

1.2   Actual and perspective planes

Klee (1956, p. 145) introduces the idea of perspective three-dimensional space within the two-dimensional graphic plane as manipulation of planes and their boundaries, lines. Accordingly the horizontal line can be seen as a surface at eye level, and two trapeziums correspond to the view from above and from below.
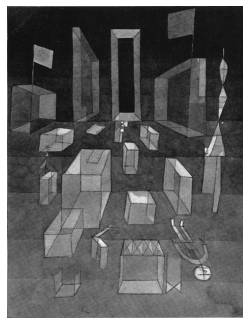

Klee
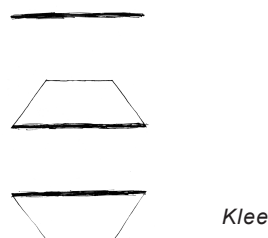
Here, operations that applyn on the graphic plane combine elements to create a higher-dimensional structure. Rules apply on planes and lines to produce this other kind of illusionistic space. The planes can be seen as actual planes or as perspective planes, horizontal or vertical. Also progression of length and weights of lines produces the same visual result.

Klee

This arrangement of elements on the plane creates the tendency for the viewer to orient himself by this new dimension. Major-minor, large or small components, brilliance-darkness, behind and front, and so the desire to bring in a third dimension gives rise to very simple rules. But what Klee has in mind is not the conventional perspective: "the value of the whole process lies solely in the possibility of checking. There is no merit to drawing in proper perspective ". Instead he establishes several viewpoints, or centers and he organizes lines and planes according to a specific graphic scale of deviation, from the horizontal or the vertical. The result is a natural deviation from static relations.
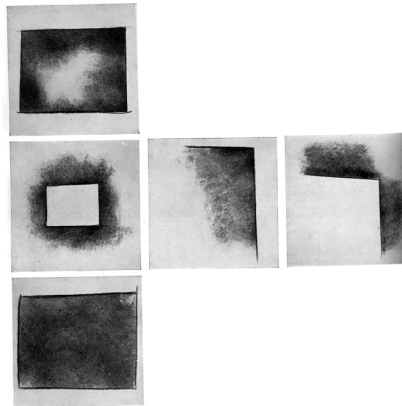

Klee

1.3   Activation of planes

The explicit reference to medial shapes, and the use of active and inactive planes, from Klee and Kandinsky is an intuitive manner to approach the ambiguity of shapes computationally. It reveals the expressive attributes of lines and linear arrangements. Lines can be seen as themselves or as boundaries of planes.

What remains implicit is that often in calculations of form we use lines, and at the end we treat their products as we prefer, in terms of dimensional properties and parts. The above becomes apparent in computations with planes on the plane. The visually active planar form is not always an economic element for computations, but its linear boundaries are. Thus, we can use them instead of the total area of the planar shape, and "activate" the planar part at the end. In other cases shapes can be activated as planes, by a figure-grounding effect that can be produced by accentuation of contrast in their linear boundaries. Klee (1956, p. 51) offers a rule of qualitative treatment of lines. He names it endotopic-exotopic treatment.The rule, which describes the movement of boundary contrast, is the following: "In handling boundary contrast, always stay on one side of the line".


Klee

This becomes particularly useful when we want to create the illusion of depth by employing the visual effect of overlapping, interpenetrating, and discrete planes. Here the boundaries become spatial and progression produces the effect of depth. The spatial character of the plane is imaginary, and depends on interpretation, but still is one of the more expressive features of two-dimensional representation that we want to treat computationally. Thus, the treatment of the linear boundaries of shapes becomes an important factor of visual expression.
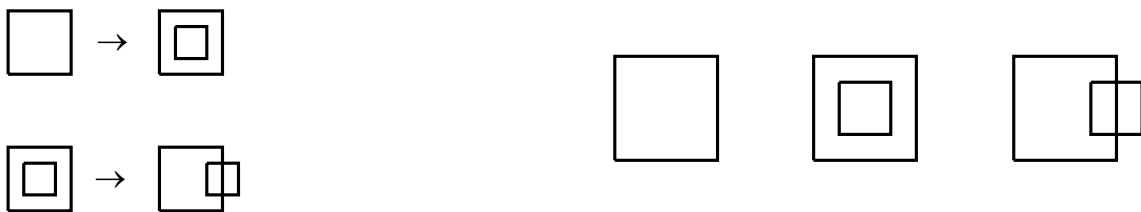
In the example the first rule places one linear square into another and the second moves it on one of the sides. Each time we assign a weight to one side of the line in a W_{12} algebra. The application of contrast on the linear boundaries of the square activates the plane and the linear shape can be perceived as a plane.

The interaction between two squares and the interaction with the graphic plane creates the visual effect of overlapping planes in three-dimensional space. This transition of dimension can be handled computationally by a compound W_{12}, W_{22}, algebra, which contains weighted lines and weighted planes on the plane.
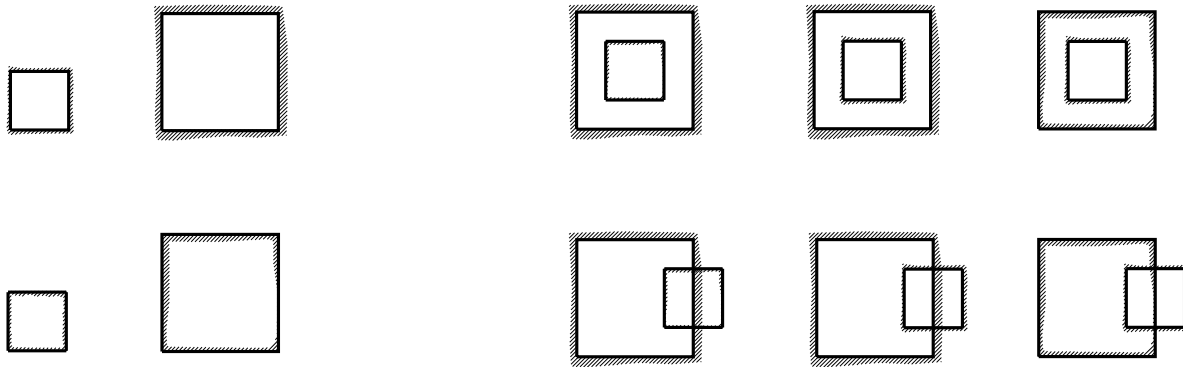
after rules apply to simple and weighted lines (U_{12}, W_{12}) to create shapes on the plane. In the example five maximal lines compose a shape in W_{12}.

The principle of endotopic-exotopic treatment can also apply to linear figures containing inter-sections.
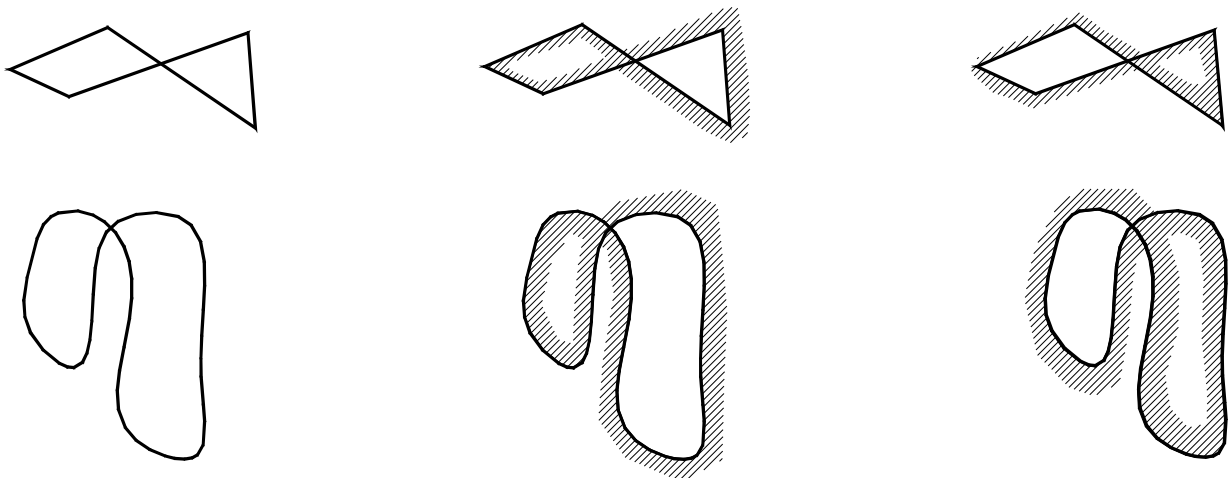
The endotopic or exotopic rule that applies on five lines creates an ambiguous visual effect. The linear shape that lies on the plane becomes an arrangement of planes, or lines, in the illusionistic three-dimensional space of the graphic plane. Emergent visual phenomena like the above exhibit vividly how our perception about visual things may change unexpectedly. Similar intellectual constructions require more clarification if they are to serve as constructive rules, but nonetheless expose the expressive power of abstract basic elements and of rules, and their intrinsic ability to generate ambiguous perceptual shifts from one dimension to another.

2.   Graphic context

When basic elements are manipulated on the plane (j=2), the graphic context can be seen as a basic plane. Kandinsky (1924, p. 114) observes that pairs of linear segments usually bound this context which in the case of painting is considered finite. Accordingly, the context in architectural composition has its own limits. In the case of a plan the shape and the boundaries of the site define this finite context for the composition, while in sections and elevations an upper, height- limit, and a lower ground-limit usually exist for the building. The shape of the site becomes in an abstract sense the graphic environment that accommodates the work and becomes part of the composition of the plan. In sections and elevations, emphasis on the verticality or the horizontality of the building provides a different context that individual architectural elements cannot alter. The shape of the finite context that is considered as a basic plane in the case of plans, section and elevations can take any form: complex many-angled shapes, circular, oval or free forms, while rectangulars appear the relatively most common in use shapes. This finite graphic context is contained into boundary lines, which possess strong perceptual significance, not only as the boundaries of a shape but also as limits of space.
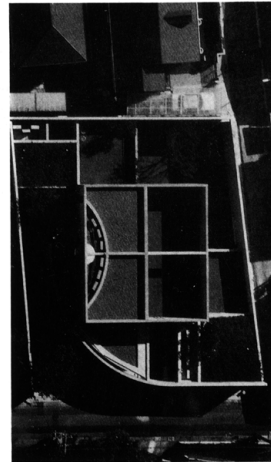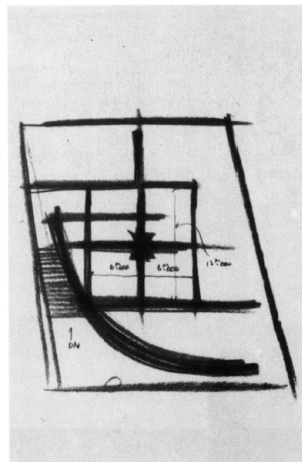
T. Ando

Each part of the finite context is individual and has its own character. The perceptual convention of "above and below", that one experiences when he stands, as a standard relationship between the three dimensional space and the body, applies on the characterization of the "top and bottom" boundary lines of the building, in section and elevation. Above and below constitute a contrasting pair, that has implications on the perception of "weights" of elements. However,

since the origin of the composition remains abstract, the application of this concept is relative. The visual convention of "right and left" amounts to the identification of the left and right limit for the composition. By convention this correspondence applies in mirrored reflection in relation to the human body. Right and left constitute the second pair of boundaries that has implications for the manipulation of elements. The graphic description of the site in plan is precisely one shape. This means that the shape and the area of the site is the same for everyone. However, the mapping of the qualities of this finite context includes the acceptance of a number of objective facts for the specific site together with the subjective interpretation and evaluation of a infinite variety of existing conditions. From this "reading of the site" architects come to descriptions that vary significantly. The process of the mapping of the site occurs in several steps, in plan and section. The sequence of these steps is unordered and has both qualitative and quantitative implications for the composition. New information may be added and subjective evaluation may change as interpretation alters. The existing conditions include the condition of the site (flat, hill, river, sea, waterfall, trees, North etc), the relation to the surrounding buildings, constructions, roads, pavements etc. Their descriptions are shapes, in plan, section and elevation.

## D.	Basic elements on the graphic context

Any form applied on the plane interacts with a finite graphic environment. The composition demands the placement of the graphic elements not only as graphic quantities or "measures" but also as qualities in interaction with this finite context. Each boundary invokes different qualities in relation to the qualities of the site. Therefore proximity of a form to the edge subjects it to different influence. The edges are distinguished from one another only by the degree of tensions in relation to the site. Each context offers two typical possibilities as regards the elements that it bears. In the first the elements lie in relation to the boundaries and its shape. In the second the elements are loosely associated with the shape of this context. The plane disappears perceptually and the elements "hover" in space. In the following examples only the square basic plane is been considered. Any other shapes of basic plane makes the visual interactions considerably more complex than the square.

## 1.	Line on plane - Relation to the border

In U_{12}, U_{22} algebra, which includes lines manipulated on a bounded plane, the possibilities for relation of a line with the linear boundaries of the plane are two. The line may be embedded on a boundary line, or it may be discrete. In the embedding case the line practically disappears since it fuses visually with the line of the boundary. In the case the line is discrete it still may touch or not touch the linear boundary of the plane. The relation of the line with the shape of the bounded plane becomes qualitatively different as the line approaches the boundaries. This relation changes even more when the line touches a boundary.
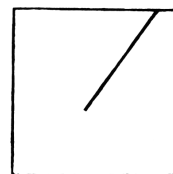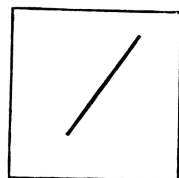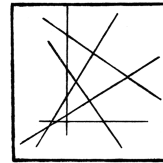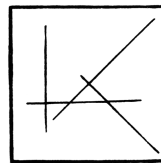

Kandinsky

A form increases its visual interaction with the shape of the bounded graphic context if is placed independently from its boundary lines. The main reason is that the shape of the boundaries and the form appear as two distinct entities that interact visually. Positioning of a form close to a specific boundary increases the intensity of the interaction with it. This relation becomes looser as the distance from the boundary becomes bigger. The interaction with the shape of the boundary changes in character when the form comes into contact with it. In the case of the line, the element and the shape become visually one shape. However, the line and the linear shape are still discrete. The following elementary arrangement of lines illustrates an extension of the above
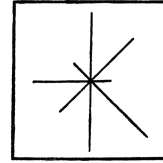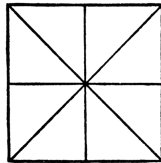

Kandinsky

In the first case the arrangement becomes a typical expression of rigidity, while in the second the same arrangement of lines detached from the boundaries acts as an independent shape in relation with the shape of the plane. In a relative example the diagonals of the left square remain centered while the vertical and horizontal move of the center. The arrangement becomes more complex. The acentric arrangement of lines of the right square intensifies the complexity while the contact of the vertical line with the upper boundary of the plane restraints the instability of the arrangement.

Kandinsky

2. Plane on plane

In the case that the line becomes a plane, the shape of the boundary and the shape of the planar element do not fuse visually into one shape but their interaction alters when the shape moves closer to the boundary. If one, or two boundary lines of the element are embedded on the boundary lines of the plane, the shape still retains its individuality but also the shape of the boundary is reinforced.
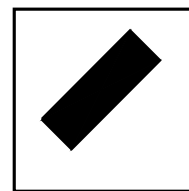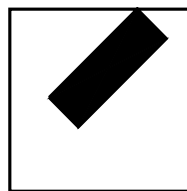
An additional observation is that forms "lying near" the boundary augment the dynamic interaction of the composition with the context, while those lying more about the center augment stability and give more "objective" character. The same visual results occur if we place a larger plane on the basic plane. For the illustration of this case the general proportions of the closed shape approach those of the basic plane.
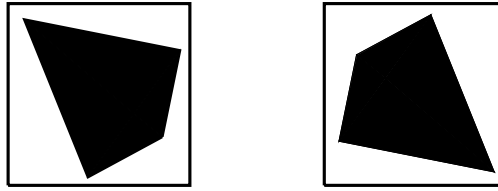
Upon these bases, further principle directions of forms can be superimposed, either remaining central or else moving away from the center in various directions. And of course, the center can be avoided altogether- the number of constructional possibilities is infinite.

CONCLUSIONS

The idea that the process of composition in design is abstract is not new. In this process combinations of immaterial graphic elements produce forms that express the schematic quality of our imaginary spatial ideas with great precision. Basic graphic elements are points, lines, and planes that combine in one, two, and three-dimensional space according to rules. Computationally, and as means of expression basic elements exhibit two different kinds of ambiguity: ambiguity of division, and ambiguity of dimension. These give rise to a series of expressive potentials and have important constructive use in design. Consequently, it becomes important for the process of composition to handle computationally both kinds of ambiguity.

Basic elements have been discussed exhaustively by mathematicians and artists. Points, lines, and planes are mathematically predefined but their graphical use demands a different approach. In their theories on non-representational composition Paul Klee and Wassily Kandinsky suggested a computational model for the intuitive treatment of basic elements. Their effort was propelled by a practical need to define equilibrium between intuition and calculation. Kandinsky and Klee accepted the notion of dimensional flux and solved it by introducing medial elements. A conspicuous similarity between these approaches and the shape computation theory is that all three are expressive systems that are concerned mainly with the process of design and less with the finished artefact. They use basic elements and rules to construct designs dynamically. Shape grammars deal with the dimensional ambiguity of basic elements in three ways:

a) use of compound algebras and weights

b) use of shape rules

c) parallel computation

The concept of an algebra that will accommodate medial shapes as well as that of finite graphic context are subjects of probable future research.

REFERENCES

Ando, Tadao 1990 Sketches, edited by Werner Blaser (Basel: Birkhauser)

Calas, Nicolas 1997 Art in the Age of Risk, and other Essays (New York: Dutton)

Chase S C 1989 'Shapes and shape grammars: from mathematical model to computer implementation' *Environment and Planning B: Planning and Design 16* pp. 215-242

Earl C F 1986 'Creating design worlds' *Environment and Planning B: Planning and Design 13* pp. 177-188

Earl C F 1997 'Shape boundaries' *Environment and Planning B: Planning and Design 24* pp. 669-687

'Euclid's Elements, the thirteen Books' introduction and commentary by Sir Thomas L. Heath 1956 (New York: Dover Publications, Inc)

Flemming U 1981 ' The secret of the Casa Guiliani Frigerio' *Environment and Planning B 8* pp. 87-96

Flemming U 1987a 'More than the sum of parts: the grammar of Queen Anne houses' *Environment and Planning B: Planning and Design 14* pp 323-350

Flemming U 1990 'Syntactic Structures in Architecture', The Electronic Design Studio, MIT Press, Cambridge pp. 31-47

Gips J 1975 Shape Grammars and their uses: Artificial Perception, Shape Generation and Computer Aesthetics (Basel:Birkhauser)

Kandinsky, Wassily 1912 'Concerning the Spiritual in Art' (New York: George Wittenborn, Inc.)

Kandinsky, Wassily 1924 'Point And Line to Plane' (Michigan: Cranbrook Press)

Klee, Paul 1956 'The Thinking Eye' (New York: George Wittenborn, Inc.)

Klee, Paul 1961 'Notebooks, Volume 2, The Nature of Nature' (New York: George Wittenborn)

Klee, Paul 1924 'On Modern Art' (London: Faber and Faber)

Klee, Paul 1953 'Pedagogical Sketchbook' (New York: Frederick A.Praeger, Inc.)

Knight T 1980 'The generation of Hepplewhite-style chair back designs' *Environment and Planning B: Planning and Design 7* pp. 227-238

Knight T 1981 'Languages of designs: from known to new' *Environment and Planning B 8* pp. 213-218

Knight T 1981 'The forty-one steps' *Environment and Planning B 8* pp. 97-114

Knight T 1986 'Transformation of the Meander Motif on Greek Geometric Pottery' *Design Computing 1* pp. 29-67

Knight T 1989 'Color grammars: designing with lines and colors' *Environment and Planning B: Planning and Design 16* pp. 417-449

Knight T 1989 'Transformations of De Stijl art: the paintings of Georges Vantongerloo and Fritz Glarner' *Environment and Planning B: Planning and Design 16* pp. 51-98

Knight T 1994 Transformations in Design (Cambridge: Cambridge University Press)

Knight T 1998 'Shape grammars' *Environment and Planning B: Planning and Design* Anniversary Issue pp. 86-91

Knight T 1998 'Designing a Shape Grammar', *Artificial Intelligence in Design '98*, p. 514

Koning H and Eisenberg J 1981 'The language of the prairie: Frank Lloyd Wright's prairie houses' *Environment and Planning B 8* pp. 295-323

Krstic D 1999 'Constructing algebras of designs' *Environment and Planning B: Planning and Design 26* pp. 45-57

Lupton, Ellen and Abbott Miller, J, editors 1991 'The ABCs of ▲■● : The Bauhaus and Design Theory' Princeton Architectural Press, Inc.

March L 1996 'The smallest interesting world?' *Environment and Planning B: Planning and Design 23*

March L 1996 'Rulebound unruliness' *Environment and Planning B: Planning and Design 23*, p. 396

Stiny G, 1975 'Pictorial and Formal Aspects of Shape and Shape Grammars', Birkhauser, Basel

March L and Steadman Philip 1971 'The Geometry of Environment' (Cambridge: The MIT Press)

Michelis, P. A. 1974 'L'esthetique de l'architecture' (Paris: Klincksieck)

Naylor, Gillian 1985 'The Bauhaus Reassassed, Sources and Design Theory' (London: The Herbert Press)

Μπιρης, Τασος Κ. 1996 'Αρχιτεκτονικης σημαδια και διδαγματα' (Αθηνα: Μορφωτικο Ιδρυμα Εθνικης Τραπεζης)

Stiny G, 1976 'Two exercises in formal composition', *Environment and Planning B 3* pp. 187-210

Stiny G 1977, 'Ice-ray: a note on Chinese lattice designs', *Environment and Planning B 4* pp. 89-98

Stiny G 1980 'Kindergarten grammars: designing with Froebel's building gifts', *Environment and Planning B 3*, pp. 409-462

Stiny G 1981 'A note on the description of designs', *Environment and Planning B 3* pp. 257-267

Stiny G, 1982 'Spatial relations and grammars', *Environment and Planning B: Planning and Design 9* pp. 113-114

Stiny G, 1986 'A new line on drafting systems' *Design Computing* 1 5-19

Stiny G, 1987 'Composition counts: A+E=AE', *Environment and Planning B: Planning and Design 14* pp. 167-182

Stiny G, 1989 'Formal devices in design' Design Theory 88 Eds S L Newsome, W R Spillers, S Finger (Springer, New York) pp. 173-188

Stiny G 1990 'What is a design', *Environment and Planning B: Planning and Design 17* pp. 97-103

Stiny G 1990b 'What designers do that computers should' The Electronic Design Studio: Architectural Knowledge and Media in the Computer Era, Eds M McCullough, W J Mitchell, P Purcel (MIT Press, Cambridge, MA) pp. 17-30

Stiny G 1991 'The algebras of design', *Research in Engineering Design 2(3)* pp. 171-181

Stiny G 1992 'Weights' *Environment and Planning B: Planning and Design 19* pp. 413-430

Stiny G 1993 'Boolean algebras for shapes and individuals' *Environment and Planning B: Planning and Design 20* pp. 359-362

Stiny G 1994 'Shape rules: closure, continuity, emergence' *Environment and Planning B: Planning and Design 21* pp. s49-s78

Stiny G 1996 'Useless rules' *Environment and Planning B: Planning and Design 23* pp. 235- 237

Stiny G 1998 'New ways to look at things' *Environment and Planning B: Planning and Design 25* pp. 71- 80

Stiny G 1999 'Commentary: Shape' *Environment and Planning B: Planning and Design 19* pp. 7-14

Stiny G and Gips J 1972 'Shape Grammars and the Generative Specification of painting and Sculpture', Information Processing 71, Amsterdam, pp. 1460-1465

Stiny G and Gips J 1978 'Algorithmic Aesthetics ', University of California Press, Berkeley, CA,

Stiny G and W.J. Mitchell 1978 'The Palladian grammar', *Environment and Planning B 5* pp. 5-18

Stiny G and March L 1981 'Design Machines' *Environment and Planning B 8* 91-104

Valery P 1924 Eupalinos: ou, l' Architecte, (Paris: Gallimard)

ILLUSTRATIONS

Images and drawings included in this study have been made by the author except otherwise noted.

All images by Klee are sketches included in the publication of his artistic notebokes (1956), under the general title 'The Thinking Eye'
(Klee, Paul 'The Thinking Eye' New York 1956 : George Wittenborn, Inc.)

All images by Wassily Kandinsky are included in his book 'Point and Line to Plane'
(Kandinsky, Wassily 1926 'Point and Line to Plane' Michigan: Cranbrook Press)

The sketch in p. 10 is by Tadao Ando (VITRA, Meditation Pavillion)
The sketches in pp. 23, 24 are by Tadao Ando (Theater on the water and Rokko Housing)
(Ando, Tadao 1990 'Sketches' edited by Werner Blaser Basel: Birkhauser)

The illustration of symmetry groups on the plane in p. 21 are by March and Steadman
(March L and Steadman Philip 1971 'The Geometry of Environment' Cambridge: The MIT Press)

The illustrations on pp. 24, 25, on the use of points in architecture are studio projects made by the students of the National Technical University of Athens, Greece. They are included as visual examples of the process of architectural composition in the book Αρχιτεκτονικης σημαδια και διδαγματα. (Μπιρης, Τασος Κ. 1996 'Αρχιτεκτονικης σημαδια και διδαγματα' Αθηνα: Μορφωτικο Ιδρυμα Εθνικης Τραπεζης), p. 36